Buyers Learning and Collusion

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In this paper we show that prices can be informative and buyers may be able to learn about the existence of a cartel. The colluding suppliers have two available strategies; to reveal their existence and enjoy high short-run profits or to avoid updating buyers' beliefs and enjoy relatively higher long-run (steady state) profits. In either case the cartel will need to satisfy the buyers' Intertemporal Incentive Compatibility Constraint in order to avoid being reported to the antitrust authorities. Our most novel finding is that the cartel will asymmetrically change its price after a shock on its costs. Not only we find that changes in prices are greater after a positive shock than after a negative one, but we also find persistence of the asymmetries, overshooting, prices initially increase by the shock and then follow a cyclical behaviour, with an alteration of the sign and lower volatility after a negative shock. Additionally we demonstrate that a temporary shock on the colluding suppliers' costs will have a permanent effect on prices. Hence, price stickiness is present in our model. Most interestingly, a positive shock is welfare enhancing since the generated steady state price will be lower than after a negative shock. Finally, we show that the number of periods necessary to converge to the new steady state price depend on the frequency of the shocks.

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PRELIMINARY AND INCOMPLETE DRAFT; Comments are welcome

I. Introduction

Academic researchers have long been exploring tools to screen or deter the existence of a cartel, both theoretically and empirically. The idea of private enforcement, buyers policing the industries, has been around for years[1]. Antitrust authorities have been using buyers' superior information for the market to prosecute or to draw their attention to the existence of a cartel. A recent example has been the bid rigging operation in the construction industry in the United Kingdom. The OFT, UK's antitrust authority, has successfully identified more than 100 companies operating a bid rigging operation from 1999 to 2006 after a buyer's report[2]. As for complaints, the OFT states "*...Complaints are vitally important in the OFT's campaign to stamp out cartels. The more information you can provide, the better equipped the OFT will be to take action...*"[3].

In this paper we introduce buyers reporting a suppliers' cartel in a dynamic setting where buyers infer suppliers' costs and competitiveness and cartel takes action against reporting. We are interested

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[1]See for example Besanko and Spulber (1990), Salant (1987) and Baker (1988).

[2]The cartel members were fined of approximately 200mn.

[3]See the following webpage $http://www.oft.gov.uk/OFTwork/competition-act-and-cartels/cartels/complain$

in answering the following questions: What determines reporting, what determines cartel's behaviour in a dynamic learning setting and are there any empirical implications?

In our model, we allow buyers to take actions against suppliers' cartel by reporting the cartel to the antitrust authority[4]. We introduce two kinds of uncertainty in the buyers' choice problem. First of all, we allow the buyers to be uncertain as to the real costs of their suppliers. We also allow the buyers to be uncertain as to whether the suppliers are colluding. In this paper we present a case with a shock-less industry and a case with shocks on costs are frequent. We show that it is possible for the buyers to update their beliefs on both the cost structure and the likelihood of collusion. Thus learning takes place in our model.

Learning takes place in the following manner; buyers have prior beliefs on the likelihood of collusion and on suppliers' costs. When a price is announced it is possible for buyers to update their information with respect to the cost realization; we assume that all other parameters[5] of the model are common knowledge. Mainly, buyers narrow down the maximum value of their suppliers' costs. Having updated their beliefs on cost, the buyers infer whether the price they had observed is rational in a competitive equilibrium and as a result they update their beliefs on the likelihood of collusion. Having updated their information set the buyers then decide whether to report the existence of the cartel. The cost of reporting are due in the same time period with reporting while the benefits are enjoyed in the next period, it takes one period for an antitrust authority to decide whether a cartel exists. The time lag generates dynamics that need to be handled.

As for the cartel members, we present two alternative strategies; an updating and *no* updating strategy. The latter provides a steady stream of profits which is lower than the the initial period's profits of the former, but higher in the long-run. To be more specific, updating generates high profits in the short-run but a low stream of profits in the long run, relatively to the *no* updating strategic. Hence, the cartel behaves self-destructively to its profitability under the updating strategy. Therefore, we identify a conflict between the strategies and the cartel's stability. The updating strategy requires a relatively smaller minimum discount factor (MDF), to be the best alternative between strategies, but at the same time the cartel requires a high enough MDF to avoid deviation. Thus, a trade-off between patience and impatience of the cartel members is identified.

In this paper we argue that it is more likely for a cartel to break down after a positive shock rather than after a negative shock. This result, though, is sensitive to the buyers' knowledge of the cost and different specifications of the shock (whether it is i.i.d.). However, the result does not reverse. To be more specific, we find that it is possible for the cartel to break down after a positive shock, but it is definitely not possible to break down after a negative shock. Hence, our paper is in line with Green and Porter's (1984) regime switching result. However, our results arise naturally from the model and this is an impact of the buyers learning process, in addition the shocks are present with respect to buyers costs rather than demand shocks, instead in their paper asymmetries are a result of the cartel members strategies.

Most importantly, we find that the cartel will price asymmetrically under different shocks. To be more specific, we find that the difference of prices after a positive shock is higher than the difference of prices after an equal in magnitude negative shock. The asymmetries persist across time and the volatility (price differences around the new steady state) of the negative shock is smaller. Furthermore, we find that the magnitude of the shock matters as larger shocks increase the asymmetry. Intuitively, a positive shock suppresses a cartel's margin and as a result the cartel will be more willing to pass-

[4]Where for simplifying reasons we assume that the antitrust will always investigate a market if a report (complaint) has been filled. Intuitively, if the public is increasingly complaining on the conduct of some companies, we believe that the antitrust authority will initiate an investigation.

[5]Such as the number of suppliers and total demand's characteristics.

through the cost increase compared to the case of a negative shock, which increases the cartel's margin. Finally, we find that price stickiness, adjustment of prices to their steady state value, depends on frequency of the shocks. To be more specific, introducing a shock in relatively shock-less industries, in one extreme, it will increase the price in a smaller magnitude, than the increase on the costs, and there is a real impact on the ss price even if the shock is temporary. On the other hand, in industries with frequent shocks, the prices overshoot and then adjust to the steady state (the price changes and becomes greater/smaller, depending on the shock, from the steady state value) and we find real effect on the prices as well.

The structure of this paper is the following: the first section presents the related literature, the next section describes the learning process and the third section, which is split into two parts, presents a shock-less industry. In the first part we characterize the no updating strategy and in the last part the updating one. We then introduce shocks to the model and describe the cartel's behaviour, identifying in that way the asymmetries. In the fourth section we introduce a simple version of an industry where shocks are hitting the industry in each period. The results on asymmetric pricing are present in this case as well. The final section concludes.

II. Literature

This paper will contribute to two strands of literature. It will contribute to the literature that examines cartel's performance and aims to identify any predictable behaviour and it will also contribute to the rocks and feathers' literature. As for cartel's performance, the only paper that presents asymmetric behaviour is the one by Green and Porter (1984), as far as we know. The authors assume stochastic demand and that the suppliers' individual prices are private information. Hence, in their model, prices may drop either because of a demand shock or a competitor dropping its prices. It is shown that it is optimal for suppliers to endorse a regime switching strategy. As a result, it is possible for suppliers to collude when demand is high and compete when demand is low. This strategy creates asymmetries with respect to the industry's response to a positive or negative shock. However, as Borenstein et al (1997) argue Green and Porter's paper will not generate persistence of the asymmetries.

Additionally, there are a number of papers that explore collusive pricing and anti-competitive behaviour in discrete-time game theoretic settings[6]. Tirole (1988), Schmalensee and Willig (1989) offer an excellent synopsis of these studies. There is also a number of papers by Motta and Polo (2003) and Spagnolo (2004), which study cartel violations in discrete time repeated game settings. Leung (1991), Feichtinger (1983), and Fent et al. (1999, 2002) have studied the dynamic (intertemporal) trade-offs between the damages generated from the offences and the costs of the antitrust authority. They have attempted to define a mix of policy variables such as prevention, treatment and law enforcement which minimize the overall social damages. Bajari and Ye (2003) investigate procurement auctions and they have developed an approach to identify and test for bid rigging operations. They have found that there is a region of bids where a competitive specification can be rejected, a region where collusion is rejected and a region where neither of the two can be rejected.

A dynamic model of optimal punishment is established in Leung (1991), where the optimal fine is calculated as a solution to an optimal control problem. Leung finds that the fine which blocks the offence is smaller than the damage caused; a finding that contradicts the result obtained from a static model of Becker (1968). Leung argues that Becker's approach does not generate, in a dynamic environment, the optimal outcome in terms of welfare. Moreover, M. Fetzi (2010) presents a case with dynamic (in)consistency with respect to an antitrust authorities (no)commitment to enforce punishment of cartels. The author finds that commitment is not always socially optimal.

[6] Asymmetries are not present in these papers.

Harrington (2004) characterized cartel pricing under an exogenous detection technology when suppliers are worried about creating suspicions for collusive behaviour. It is shown that balancing concerns about sustaining the stability of the cartel with concerns about avoiding detection, may force the cartel to either gradually raise the price to its steady-state level or gradually raise the price and then have it decline to its steady-state level. Antitrust laws may have a negative effect as they reinforce cartel stability and in this way allow for higher cartel prices.

The closest paper to ours is the one of Harrington and Chen (2006). They distinguish collusive pricing patterns when buyers may detect the presence of a cartel. Buyers are assumed to become suspicious when they observe that prices are anomalously. The authors examine cartel pricing, find a price path, when the buyers might detect a cartel. They conclude that the cartel price path comprises of two faces. During the transitional phase price is generally rising and is relatively unresponsive to cost shocks. During the stationary phase price responds to cost but is much less sensitive than under competition or simple monopoly; a low price variance may then be a collusive marker. Moreover, compared to when suppliers do not collude, cost shocks take a longer time to pass-through to prices. In their model buyers are assumed to become suspicious when the prices behave anomalous. However, their result seem sensitive to the shock's specification and buyers' beliefs. To be more specific, if the buyers anticipate a large price variance (in a competitive regime) then their results might reverse. Furthermore, as we are going to show if the shock is i.i.d. and the buyers observe the cost shock with a noise then the cartel will have an incentive to increase the variance of the price, relative to the competitive, and profit from the increased variance.

Harrington and Chen's conclusions are tested by Bolotova, Connor, Miller (2008) and they find support for their conclusion on the lysine cartel but not on the citric acid. Abrantes-Metz, et al (2006) find that in a bid-rigging conspiracy the variance of prices decreased, they test the predictions of this model into the gasoline market and they find no evidence for collusion.

In contrast to Harrington and Chen (2006), in our model the buyers do not condition their beliefs on the prices but instead on the costs, which might be more appropriate in some industries (as the retail gasoline one). Most importantly, the authors do not examine the incentives of the buyers and as a result do not examine any strategic interaction between the market participants. However, as we are going to show there is always the possibility for the cartel to reveal its existence and choose a price such that the buyers are just indifferent from reporting, satisfying buyers' Incentive Compatibility Constraint. Furthermore, we examine price asymmetry something that is not explored by Harrington and Chen.

As for the rocks and feathers' literature, a number of papers have found an empirical stylized fact. Peltzman (2000) examines prices in more than 200 industries and finds evidence of asymmetric changes in most of the sample. Furthermore, S. Borenstein, et al. (1997) was the first paper, to our knowledge, that identified this empirical regularity in the gasoline industry. The authors estimate the asymmetries to the whole chain of production. Johnson (2002) finds that diesel prices respond faster and less asymmetrically than gasoline prices. Eckert (2002) tests the hypothesis that Edgeworth cycle equilibria can produce asymmetric changes and finds some empirical evidence. However, the Edgeworth cycle theory, used by Eckert (2002) and Noel (2007), predicts that prices should behave independently of wholesale costs something that is not supported in other studies. In another series of papers researchers investigate whether asymmetries arise in other industries as well. For instance, Goodwin and Holt (1999) and Goodwin and Harper (2000) estimate asymmetric changes in the U.S. beef and pork industries, and O Brian (2006) estimates asymmetric adjustment in interest bearing deposit accounts.

The rocks and feathers' literature suggests that capacity constraints, see Maskin and Tirole (1988),

and search costs, see M. Tappata (2009), H. Yang and L. Ye (2008)[7], can explain the asymmetric responses to shocks in a competitive setting. However, the gasoline stations are located quite close to each other (some of the times located within 50 meters of distance) and nowadays search costs have declined, which means that the search costs argument might not be applicable always in the retail market. Furthermore, the capacity constraint models might not be applicable to a retail industry, it has been suggested in the literature that the retail part of the industry is not investing on capacities. As for papers that refer to collusive behaviour, Green's and Porter's (1984) paper has been widely referred as a model that can explain asymmetries, as it was mentioned before. Their model though is not inappropriate to explain the persistence of the asymmetries that have been found by empirical work. Additionally, in that paper the asymmetries are generated by the cartel members imperfect monitoring ability and their most important assumption is that the prices are privately owned which is not appropriate to the retail gasoline market. As we are going to show, in our model asymmetries arise due to the asymmetric information between suppliers and buyers and the learning process. The next section presents the model.

III. Buyers Learning

In our paper we assume a linear demand function, $P = a - bq$ where a is the maximum willingness to pay and b the responsiveness of buyers to a change in quantity. We will also assume that the suppliers are faced with a constant Marginal Cost (MC) that is realized from a continuous uniform distribution $c \sim U(0, C)$ in period zero only, hence an industry without shocks. The suppliers are also faced with a fixed cost F. Assume that buyers know the value of F and they know that the MC comes from a continuous uniform distribution but do not observe the realized value. Thus, the buyers are not fully informed and need to make inference on the possible realized cost of their suppliers[8], $c_b \sim U(0, C_{max})$. C_{max}, with $C_{max} > C$ but they know how that the suppliers are choosing quantities (competition à la Cournot, which is common knowledge). The buyers however do not know whether their suppliers are behaving competitively or whether they are colluding. Hence, learning is present with respect to both the cost realization and the competitiveness of the suppliers (two sources of uncertainty; competition and cost).

Let the timing be the following; a new market opens (birth)[9]. The firms decide to enter non-cooperatively based on the realization of the MC, which is the same to everyone. The entry takes place in period zero only, this is a simplifying rather than a necessary assumption[10]. The firms will enter if it is profitable, the fix costs assures a non-negative profit for the entrants. After entry we assume that the suppliers have an exogenous opportunity to form a cartel. As for the buyers, they form beliefs on the MC prior the entry. When the entry is observed the priors are updated. Let the timing within a given period be the following, the suppliers announce a price[11] the buyers update their beliefs and choose whether to report or not. If the buyers report the antitrust authority always investigates and dissolves the cartel permanently. We believe that the last assumption is not an irrational assumption since the members of an antitrust authority do initiate investigations in markets when the public has been complaining on a particular industry[12]. We also assume that there is no

[7]Other papers include work by Lewis (2011) and Cabral and Fishman (2006) among others.

[8]Alternative, we can think that the buyers observe that the companies have payed the fixed costs which means that there is a price which generates trade and as a result $C_{t=0,max} \leq a$

[9]This is a necessary assumption for the no-stochastic industry since, as we are going to show, any price change will result into buyers updating their beliefs for the likelihood of collusion (which in this case will be equal to one. This assumption, however, is not necessary for an industry that exhibits stochastic shocks on its costs.

[10]Relaxing this assumption will only result to restrain the collusive profits up to the point where a potential entrant is indifferent from entering, hence to deter entry.

[11]Based on the realized production choices.

[12]The cost of not investigating will be higher than the cost of investigation. Since if they do not investigate the public will

penalty imposed by the antitrust authority. The las assumption is taken in order to further simplify the analysis[13]. Furthermore, in this paper we are mainly interested to examine the deterrence power of the buyers in an environment of asymmetric information.

As for buyers updating process, this will take place with respect to the upper and lower bound of the cost and it will be updated from the information provided by the realized prices. The following corollary presents how updating takes place.

Corollary 1:

The maximum cost cannot be greater than the realized price in a shockless industry.

Proof: Assume $C_{t,max} \geq P_t$ which means that the probability of the price being smaller than the cost is zero ($Pr(P_t < C_t) = 0$, where Pr is for probability). This holds since the suppliers $q_i > 0 \Rightarrow \Pi_i > 0$, where Π is the profit of i, otherwise the suppliers would have chosen $q_i = 0$. Furthermore, $Pr(P_t \geq C_t) > 0$ as a result the buyers still do not know the realized cost of their suppliers but they know that $C_{t,max} = P_t$. Assume, $C_{t,max} < P_t$ but at the same time $Pr(C_{t,max} < C < P_t) = 0$ since otherwise the prior beliefs $C_{t,max}$ would have been irrational.To be more specific, the prior belief of the cost will be formed based on the observed parameters, a,b and n. Furthermore, the max value of the cost will be the one that generates profits smaller or equal to the fix costs divided by the number of companies. $\Pi_i = (a - bnq_i - c)q_i = \frac{\lambda F}{n}$ where λ is a parameter that measures the profitability available to the firms from a benchmark entry game[14]. This means that $C_{max} = a - (n+1)\sqrt{\frac{(n+1)F}{n}}$, having substitute for the Cournot quantity and for $\lambda = 1$ its maximum value. Hence, $Pr(C_{t,max} < C) = 0$.

QED

The buyers at any given period know that the maximum cost cannot be greater than the previous period's price. However, the buyers will be able to infer the true cost if the suppliers are competing, while they will be able to update the upper bound of the cost if the suppliers are colluding. This is presented in the fist lemma that follows.

Lemma 1:

In a shockless industry the price is fully revealing under competition and partially revealing under collusion, with respect to the marginal cost of the supplier. The updating of the cost is:

$$\begin{cases} C_{true} = \frac{P_t(n+1)-a}{n} & \text{if competition,} \\ E_{t+1}(C) = \frac{P_t}{2} & \text{if collusion and } P_t \leq C_{t-1,max} \\ E_{t+1}(C) = \frac{C_{t-1,max}}{2} & \text{if collusion and } P_t > C_{t-1,max} \end{cases}$$

Proof: If there is competition $P^c = \frac{a+nC}{n+1}$ which means that $C_{true} = \frac{(n+1)P^c - a}{n}$. Hence, the price is fully informative. While if there is collusion then the observed price can be anything between

believe that the antitrust authority is not doing its job and the likelihood for the antitrust authority to be punished by politicians increases (change of the head of the antitrust authority or decrease of government funding).

[13]Relaxing this assumption will increase the difficulty for the cartel formation (a higher minimum discounted factor, as defined by the trigger-Grim strategy) will be required. But it will not influence the cartel behaviour in our model after its formation (at least in our model).

[14]The suppliers will enter up to the point where their profits are nonnegative. Hence, the entrants will have a positive return from the deterrence effect of the fix costs. The maximum value of their profits will be equal to $\frac{F}{n}$ and the minimum is zero.

the collusive benchmark ($P^m = \frac{a+c}{2}$) and the competitive price. Furthermore, the lower bound (competitive price) tends to zero as the MC goes to zero and number of suppliers goes to infinite. Hence, the price can not be fully informative. However, if $P_t^m \leq C_{t-1,max}$ then the upper bound of the possible cost will be equal to the price, see corollary 1, and as a result $E_{t+1}(C) = \frac{P_t}{2}$. If however, $P_t^m > C_{t-1,max}$ then the price is not informative, neither the lower nor the upper bound is updated.

QED

The intuition is that under competition the price is fully informative since all the parameter are known. In the case of collusion, the expected cost is half the last period's price or the $C_{t,max}$. This is a consequence of our assumption that the cost distribution is uniform and our result that price is partially informative under collusion.

The above analysis assumes that the buyers know whether the suppliers collude. If this is not the case then the buyers cannot distinguish whether the price is fully or partially informative with respect to the cost. In order to incorporate this uncertainty of the buyers we will assume that the buyers have prior beliefs on the likelihood of collusion. Let denote this belief as ξ, the probability that a cartel exists and $1-\xi$, the probability of competition. Now we are able to characterize the buyers' expected cost of their suppliers (the updated beliefs).

$$E_{t+1}[C_b] = \xi_t \frac{P_t}{2} + (1-\xi_t)\frac{P_t(n+1)-a}{n} \tag{1}$$

The above equation states that the expected cost is a weighted average on the information provided under competition and under collusion. This is a consequence, of both the first corollary and the first lemma. Furthermore, the upper bound of the cost is going to be updated based on $E_{t+1}[C_b]$. Thus we have the following condition.

$$C_{t+1,max} = min\{E_{t+1}[C_b], C_{t,max}\} \tag{2}$$

The above condition states that the maximum value of the cost (upper bound) is going to be updated only if $E_{t+1}[C_b] < C_{t,max}$. This condition is necessary since it might be the case that the cartel will choose a price that will not update the upper bound of the cost, but as we are going to show it will update the beliefs of the buyers on whether a cartel exists. The most challenging part for the buyers is to infer whether or not a cartel exists. We assume that given buyers' prior information on the cost, ξ will be equal to its prior, ξ_{prior}, if the price is the same across time (shock-less industry) and equation (1) is less or equal to the initial (prior) belief of the maximum cost $EC_t[C_b] \leq C_{max}$, otherwise $\xi = 1$. The last assumption suggests that if the inferred costs are smaller than the prior beliefs then the buyers will believe that there was no change on the competitiveness of the industry. The assumptions, on ξ, are summarized below:

The ξ will be updated if the prices change across time and if the expected cost, $E_{t+1}[C_b]$, is greater than the observed price.

$$\xi_t = \begin{cases} \xi_{prior} & \text{if } P_{t+1} = P_t \text{ and } E_{t+1}[C_b] \leq C_{t,max} \\ 1 & \text{if } P_{t+1} \neq P_t \text{ and/or } E_{t+1}[C_b] > C_{t,max} \end{cases} \tag{3}$$

The last part of the above conditions is actually easy to prove. If the prices changes across time in a shockless industry then the suppliers should be colluding. There is no reason why prices should be changing under competition, linear demand and marginal costs in a no-stochastic industry. Furthermore, if the expected cost is greater than the prior then this will only be a consequence of a collusive behavior, see the first corollary. For all other cases the buyers do not know whether a cartel exist as or not and as a result we assume that ξ is not updated.

The first condition suggests that there is no updating on the likelihood of collusion if prices are constant across time and/or the updated beliefs on cost are lower than the initial beliefs on the maximum cost. In the case where the price changes across time that will mean only one thing, the suppliers are colluding in a shock-less industry. However, this will not be the case in an industry with frequent cost shocks and as a result care needs to be taken. Furthermore, if the updated expected cost is higher than last period's beliefs on the maximum cost then ξ will be equal to one, suppliers are colluding. This is the case when the inferred cost cannot be rationalized as a competitive outcome. In other words, for any given C_{max} there is an upper bound for a Cournot price $P^c_{max} = \frac{a+nC_{max}}{n+1}$. If the price ($P_t$) is higher than the maximum Cournot price (and as a result $E_{t+1}[C_b] > C_{t,max}$) then ξ will be equal to 1. The buyers will know that the price observed cannot be set in a competitive environment for any cost realization. Finally, if the prior beliefs on the likelihood of collusion are updated ($\xi = 1$) then the cost beliefs need to be revised, since they were structured based on the assumption that ξ equals its prior value. To sum up, updating takes place in three steps:

A. Update Cost beliefs.

B. Update Likelihood of Collusion (ξ_t).

C. If $\xi_t \neq \xi_{t-1}$ repeat step A with the new value for ξ_t.

However, we still need to show that the cartel's own ICC (collusion needs to be profitable) is satisfied. By comparing the profit functions between collusion and competition, and using the fact that the cartel has no incentive to price above the benchmark collusive price (P_m) we get the following conditions[15]:

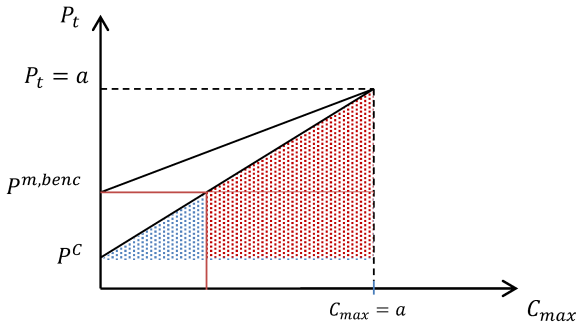
$$P_m > \frac{a+nc}{n+1} \tag{4}$$

$$P_m \leq \frac{a+c}{2} \tag{5}$$

The above conditions need to be met irrespective of the strategy of a cartel (updating or no-updating). Notice that the maximum price that a cartel can set is the upper bound defined in equation (5). This equation changes by a fraction of $\frac{1}{2}$ from a unit cost change, while the Cournot price changes by $\frac{n}{n+1}$. The regions of collusive and competitive prices are depicted below:

Figure 1: Collusive vs Competitive prices

[15]Using $\Pi_m > \Pi_c$, which is equivalent to $(P_m - C)(a - P_m) > (P_c - C)(a - P_c)$



In the horizontal axis we have the buyers' maximum anticipated cost under different priors and on the vertical axis we have the price. The shaded areas correspond to a price that can be rationalized as competitive by the buyers, given any value of C_{max}. In all of the cases the cartel, in order to avoid updating buyers' beliefs on ξ (likelihood of collusion), will need to set its price at most equal to the maximum anticipated Cournot price, for given beliefs on the max cost. The red region is the set of cost realizations (true cost) that might possibly generate a benchmark collusive price that can be rationalized as Cournot. For values below that region (blue shaded area) there is no support on true cost that can generate a benchmark collusive price and be rationalized as Cournot. This is in line with P. Bajari and L. Ye's (2003) finding that there are some regions (in their case with respect to the bids distribution) where either competitive or collusive behaviour can be rejected and others where it can be inconclusive. As for the cartel, if the maximum anticipated cost belongs to the blue region then it can set at most the max Cournot price, which means that if the realized cost equals to the anticipated maximum cost then the collusion is not possible without updating buyers' beliefs (since $P_t^{m,benc} > P_t^c(C = C_{max})$). The starting point of the red region will be found by setting $P^{m,benc}(C = 0)$ equal to the Cournot price at max cost $P^c(C = C_{max})$. The red shaded area is for values of $C_{max} > \frac{(n-1)a}{2n}$. Notice also that the set of possible collusive prices are greater than the set of possible Cournot prices (regions below the line of $P^{m,benc}$ and P^c). Finally, a conclusion is that a collusive behaviour can be much easier be traced at very low prices (generated from low costs), conditional to low priors on the cost, rather than high prices.

The next task is to describe the deterrence power of the buyers to the cartel's behaviour. We assume that the antitrust authority will always investigate if the public (buyers) report (complain) on the behaviour of their suppliers. Assume that buyers have a cost of reporting V. If the buyers report in period t they will enjoy the benefits of competition in the next period for at most one period[16], given that a cartel was found and broken down by the antitrust authority. A buyer will not report if the consumer surplus received in two periods from not reporting is greater than reporting. The fact that the benefits of reporting are enjoyed in the next period generates an intertemporal problem for the buyers. The condition for buyers *not* to report is provided below.

$$CS_t(P_t) + CS_{t+1}(P_{t+1}) \geq CS_t(P_t) - V + CS_{t+1}(P_{t+1}^c)$$

Notice that the first terms in either side of the inequality cancel out. Hence, the decision of the buyers to report does not depend on current CS and this is because of the benefits being enjoyed in the next period. The last term at the rhs is the expected CS when buyers are certain that competition is imposed. All terms dated at $t+1$ are conditional on the information available to the buyers on the maximum cost and the beliefs for the likelihood of collusion at t. Notice that it is possible for

[16]We are implicitly assuming that the buyers are short sighted. Alternatively we could have assumed overlapping generations of equal size that live for two periods with identical preferences.

the buyers to wrongly report a competitive market. This, might be the case if the expectation on the max cost was low enough but the realized price supports a higher realized cost. As a result, if it is incentive compatible the buyers will decide to report. This will be more formally stated in a latter stage.

The above condition describes the Intertemporal Incentives Compatibility Constraint (IICC) of the buyers. The buyers will report if the CS generated from the price in $t+1$ is smaller than the CS under competitive price in $t+1$ after deducting the cost of reporting. The choice of reporting is taken in period t and as a result we need to take expectations on the above equation.

$$E_t CS_{t+1}(P_{t+1}) \geq E_t CS_{t+1}(P^c_{t+1}) - V \tag{6}$$

The lhs can be decomposed further more. It will be equal to the likelihood of collusion (ξ) multiplied by the expected CS under collusion plus $1-\xi$ times the expected CS under competition. We are also going to assume that the cost of reporting is a function of the CS at time t ($V = vCS_t$). Intuitively, the cost of reporting corresponds to the CS that the buyers are willing to sacrifice today in order not to report. Being a fraction of CS means that as quantity (price) increases (decreases) the buyers become more willing to report, since they will benefit the most. An example might be the users of social networks (e.g. Facebook), heavy users might be more willing to report a complain (and more likely to trace an issue) to consumers protection authority rather than a low user. The heavy users (that receive higher utility) will be more willing to report since any marginal increase on the CS will have a greater impact. Similarly, a large buyer of an industry might be more willing to report, than a small buyer, since its benefit from a switch to competition will be higher.

As for the benefits of reporting, buyers need to form expectation on the actual value of the benefit which arises in the next period. Furthermore, with the use of simple algebra and the fact that the CS under linear demand can be expressed as $\frac{(a-P)^2}{2b}$, we generate the following condition (from inequality (6)):

$$E_t[(a - P^m_{t+1})^2] \geq E_t[(a - P^c_{t+1})^2] - \frac{v(a - P_t)^2}{\xi}$$

As we can see the P^m_{t+1} appears in the left hand side of the above inequality. This is an anticipated by the buyers collusive price. If the cartel chooses the above price (P^m_{t+1}) then it is not necessary that it will reveal itself to the buyers. As we are going to show in the *no* updating case (where the cartel chooses not to reveal its existence) the cartel will choose a price that is can be rationalized as both collusive and supported by a Cournot outcome (shadowed region of figure 1) and at the same time satisfy the above condition. Furthermore, we can further decompose the above inequality by using the variance formula[17]. Notice that in a shock-less industry the variance of the competitive price equals to zero. Furthermore, the 1st lemma establishes the IICC with hold with equality.

Lemma 2:

> The IICC is satisfied with equality by the cartel members if the cost of reporting in not prohibitively high.

Proof: If the cost of reporting (V) is such that the rhs of (6) is always greater than the lhs then

[17] $E_t(X^2) = Var(X) + (E_t X)^2$

the buyers will never report (the inequality can never hold). If however the cost or reporting is low enough then setting a collusive price that generates higher values for the lhs of the above equation (high CS under collusion) cannot be optimal for a cartel since it can increase its price (increase profits) and avoid being reported at the same time. As long $P^{B,ben} > P_{t+1}$, where P_{t+1} is defined by (6) and $P^{B,ben}$ is the benchmark collusive price, the cartel will be able to raise its price $P_{t+1} < P'_{t+1} < P^{B,ben}$ increases its profits,$\Pi(P_{t+1}) < \Pi(P'_{t+1}) < \Pi(P^{B,ben})$ and avoid being reported. Hence, the cartel has no incentive to set a price that will generate a higher CS at the lhs than the rhs of (6) (it will increase the price).

Assume that $\bar{P}_{t+1} = P_{t+1}$ is the price that generates equality for (6). Choosing $P_{t+1} > \bar{P}$ that will generate lower CS than the value of the rhs and as a result the buyers will report. If the buyers report the antitrust authority will investigate the market and dissolve the cartel with probability one. For the cartel members a break down of the cartel is not an option since by the argument of deviation the cartel will never be formed (if the cartel has a predetermined finite date of break down then it will break down from period zero). Hence, the above equation needs to be satisfied with equality.

QED

The intuition is that the cartel has an incentive to provide as small CS as possible in order to maximize the collusive profits[18] but at the same time to satisfy the incentive of the buyers not to report. As a result equation (6) needs to be satisfied with equality. Furthermore, from now on we are going to assume that the cost of reporting is low enough so that the cartel members need to satisfy the IICC.

The variance of the collusive price in the next period needs necessarily to be zero as well, since a rational buyer will anticipate that the cartel will satisfy the above equation and as a result there will be an unique price[19]. Thus, the above condition becomes:

$$(E_t[a - P^m_{t+1}])^2 = (E_t[a - P^c_{t+1}])^2 - \frac{v(a - P_t)^2}{\xi}$$

Substituting for the expected competitive price, equation (1):

$$(E_t[a - P^m_{t+1}])^2 = \frac{(2(n+1-\xi)a - P_t\{2(n+1) - \xi(n+2)\})^2}{4(n+1)^2} - \frac{v(a - P_t)^2}{\xi} \tag{7}$$

Having established that the cartel will need to satisfy the buyers IICC, equation (7), our next task is to examine the cartel's available strategies. The cartel has two possible intertemporal strategies that can satisfy the IICC; set an unique price across time or set an initially high price and then decrease its price across time until either it reaches a steady state or the cartel breaks down (the optimal strategy and more details on the cartel strategies will be given later on). The unique price corresponds to the choice that does not allow any updating of buyers beliefs, on the existence of a cartel (ξ) or the cost. Hence, the cartel will profit from the ignorance of their buyers. However, this strategy might enforce the cartel to set a low price relative to the benchmark collusive. An alternative strategy is to set a

[18]Since the cartel profits are negatively related to the CS.

[19]The price is not a random variable. In the appendix we show that relaxing the zero collusive variance assumption will not influence our results. The intuition is that the law of motion (IICC) will be redefined adding also the variance term but the incentives of the cartel and the asymmetries that will be generated will still hold. The asymmetries depend on the updating process rather than the IICC and the incentives of the cartel is always to satisfy the buyers IICC whatever form it takes.

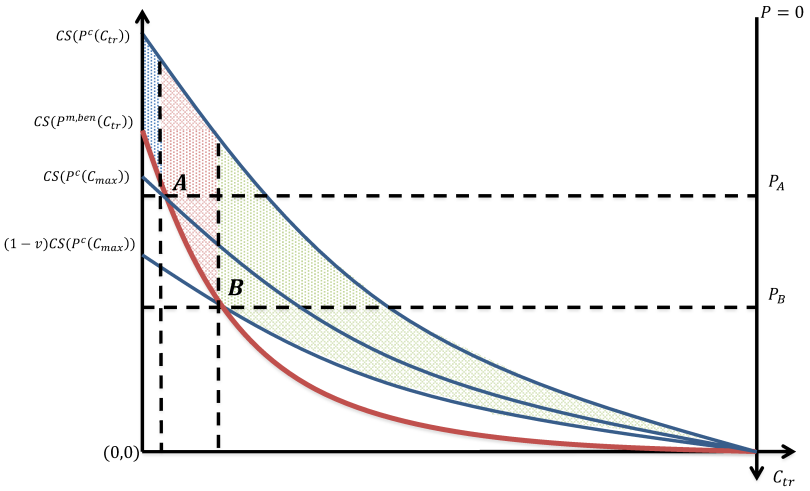
high price in period zero, allow for updating on the likelihood of collusion (with $\xi = 1$) and decrease the price across time in order to satisfy the IICC, gradually updating the beliefs on the realized cost as well. The last strategy will generate a long run (steady state) price that will be necessarily smaller than the one under no updating. In the latter strategy, the cartel can no longer profit on the buyers ignorance of ξ but it can profit from their IICC, from the incentives not to report and the uncertainty of the competitive price (true costs are unknown). Which of the two strategies are optimal will be investigated later on. Imposing $\xi = 1$ for the updating equation we generate two IICCs.

$$E_t P^m_{t+1} = a - \frac{\sqrt{\xi_t}}{2(n+1)\sqrt{(v+\xi_t)}}[2a(n+1-\xi_t) - P_t(2(n+1) - \xi_t(n+2))] \tag{8}$$

$$E_t P^m_{t+1} = a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t)^2 - v(a-P_t)^2} \tag{9}$$

The above equations relates future prices to current, in a collusive industry. If the cartel chooses P^m_{t+1} then it will generate an expected future CS (that might be collusive of competitive) that makes the buyers just indifferent on reporting. Furthermore, when the buyers observe $P_{t+1} = P^m_{t+1}$ this can be rationalized as both competitive and collusive. The following figure presents the buyers incentives under different cost realizations.

Figure 2: Buyers Incentives



In the above figure we have at the left hand side the CS on the vertical axis, the cost on the horizontal axis and prices at the vertical in the right hand side. We have plotted the competitive CS under the realized costs (curve that corresponds to $CS^c(P(C_{tr}))$, where tr is for true) which we have assumed that is greater than the CS under the maximum possible competitive CS (the curve that corresponds to the $CS^c(C_{max})$). We also plotted the benchmark collusive CS and the maximum anticipated competitive CS reduced by the cost of reporting, $(1-v)CS^c(P^c(C_{max}))$. It is straightforward to show that the CS is a convex function of the cost realization. Furthermore, in the graph we have plotted $CS(P^{m,ben}(C_{tr}))$ being greater than the maximum competitive CS, this is not necessarily the case but it is possible for some priors on the cost to generate this result. By assuming that the buyers have

high initial priors on the cost we have generated an extra region (blue)[20] , which otherwise might not have existed, in order to present all possible scenario. To be more specific, the intercept of the $CS(P^{m,benc}(C_{tr}))$ at the vertical axis might have been smaller than the intercept of the $CS(P^c(C_{max}))$. Finally, we have introduced on the vertical axis at the rhs the price in an inverse way, in order to present the effect on price from changes on the CS (it is also straightforward to show that the price is inversely related to the CS).

The above figure generates three regions of interest. In the blue region the cartel can set the benchmark price, which is rationalizable as competitive, and avoid being reported or update buyers beliefs on the likelihood of collusion. The cartel is avoiding reporting because it is not incentive compatible for the buyers to report, since the CS under reporting is lower than the CS generated by the benchmark collusive price. For this region to be generated we need the priors of the buyers to be are either high enough, with respect to the max support of the cost, or the probability of collusion is low enough. The red region corresponds to a cartel setting the benchmark price allowing in that way updating on ξ but on the other hand avoiding being reported as it is not incentive compatible. Mainly because, the CS that is generated under the benchmark price is higher than the expected competitive CS under competition with a maximum cost realization. The cartel, though, does have the choice of setting a price smaller than the one that generates $CS^c(P^c(C_{max}))$ and as a consequence avoid updating ξ, the optimal choice will be characterized in a later section. The last region, the green one, presents a case where the cartel needs to set a price that is smaller than the benchmark collusive in order to avoid been reported. In this case, the price needs to be set in a way that satisfies buyers IICC and as a consequence the green region is bounded below by the $CS^c(P(C_{max}))$ curve. As for points A and B, those correspond to threshold points that generate a switch to the pricing strategy of the cartel. These points are found when benchmark collusive CS crosses the CS under C_{max} (point A) and when it crosses the curves that incorporates the CS under C_{max} and the cost of reporting.

There are a number of conclusions that can be derived from the above figure. First of all, low prices do not necessarily suggest a competitive behaviour of the suppliers (blue region) and similarly high prices do not suggest collusion (green region), this is a straightforward result that verifies Industrial Economists beliefs on the performance of a market. However, the interesting result is that we should never observe reporting when a cartel exist, since it will not be incentive compatible for the buyers. Furthermore, the threat of reporting can be a curse and a blessing for the buyers. A blessing since the cartel is bounded to price lower than the benchmark one (in all regions except the blue one) generating higher CS for the buyers and implicitly compensating the buyers in order not to report. At the same time, it is a curse because the buyers can do nothing to terminate collusion (not incentive compatible). This suggests that market participants (buyers and suppliers) might share a common well hidden secret, the existence of a cartel, that might not be verifiable by the outsiders.

On the other hand, reporting is possible under competition if the cost realization and the beliefs on the likelihood of collusion (ξ) is high enough and the cost of reporting is low enough. This generates an IICC with higher expected utility under reporting. The driving force is the buyers uncertainty. A high price is more likely to be collusive and as a result the buyers are more willing to report. The following argument will be further explored in the next section.

For the analysis that follows we will simplify the notation a bit further and set $\gamma = \frac{v}{\xi}$. The γ parameter captures two opposing forces that influence the cost of reporting. First of all, changes on ξ have a negative impact on the cost of reporting. Buyers are more willing to report if ξ is close to 1, if the buyers are very suspicious of their suppliers colluding. On the other hand, the v parameter influences positively the cost of reporting. An increase on the willingness of buyers to avoid reporting

[20]The regions will be explained in a while.

(v) has as result to increase the reporting costs. Overall, if the parameter γ increases the cost of reporting increases as well. Furthermore, the remaining terms that depend on ξ correspond to the effect of uncertainty on the expectation of the buyers. As a result this simplification allows us to separate the effects of ξ on cost of reporting and the uncertainty on future CS. The next sections solve for prices under different strategies and characterize necessary and sufficient condition for each strategy to be preferred from the competitive status.

A. Collusion, no updating strategy in a Shockless Industry

The setting of a shockless case can be relevant to industries with infrequent or negligible shocks on the cost. First of all, notice that the IICC presented in the previous section need to be satisfied by the cartel at least with equality in order to avoid being reported[21]. This means that P_t can only be the collusive price $P_t = P_t^m$[22]. Hence, the cartel needs to satisfy a price law of motion defined by equation (8) and its own incentives constraints, equation (4) and (5). The first strategy that we examine is the *no* updating one. The no-updating price will be found by setting $E_t[P_{t+1}^m] = P_t^m$ in equation (8).

$$P_{\sim u}^{m*} = \frac{2a[(n+1)(\sqrt{1+\gamma}-1)+\xi]}{[2(n+1)(\sqrt{1+\gamma}-1)+\xi(n+2)]} \tag{10}$$

Where the symbol $\sim u$ is for no updating and that ξ will be set to the prior beliefs of the probability that the suppliers are colluding. It is trivial to show that the above price is going to be less than a. Furthermore, the above price is independent from the cost. This is because the buyers are conditioning their beliefs on the costs and the cartel needs to endogenize their beliefs and set a price to avoid updating. The price does depend positively on the cost of reporting, negatively on the parameter γ and negatively on the prior beliefs on collusion (it is straightforward to show the last argument by taking the derivative of (10) with respect to ξ). Thus, a change on the cost of reporting (legislative or with respect to antitrust authorities credibility and successfulness) or the prior beliefs will influence the price positively and negatively respectively. For prices lower than (10) there is no updating on ξ, assuming max priors are higher from the expected cost that is generated by equation (10). Furthermore, the IICC is satisfied by equality thus any price below (10) will make the buyers better off by no reporting and as a result a price that is smaller than (10) is no-updating as well. However, for higher prices updating cannot be restricted (the IICC will generate reporting and as a result updating on cost and ξ) and as a consequence there will be no collusion with a no-updating strategy. Furthermore, the buyers will report whenever the price is larger than (10). As a result we can now identify reporting under competition.

Corollary 2:

If P^c is greater than $P_{\sim u}^{m*}$ then the buyers will report to the antitrust authority.

Proof: Since equation (10) is constructed such that the buyers will not report any price greater than (10) should generate reporting. However, we still need to show existence for the cost that will generate a higher price (than 10). By setting equation (10) equal to the competitive price ($P^c = \frac{a+nc}{n+1}$). This generates the following condition:

[21] Otherwise the cartel will be reported and dissolved by the antitrust

[22] The suppliers know whether they are colluding

$$C_{true} = \frac{a[(n+1)(\sqrt{1+\gamma}-1)+\xi]}{[2(n+1)(\sqrt{1+\gamma}-1)+\xi(n+2)]}$$

Hence, if the cost is larger than the rhs of the above condition that will generate a competitive price higher than (10) and as a result reporting from the buyers.

QED

The above corollary shows that it is possible under competition for reporting to take place. The intuition is that a high cost has a low probability of realization and because of the uncertainty the buyers are willing to report[23]. This result might explain in same cases buyers actually complaining for the high gasoline prices where actually the suppliers are behaving competitively. Actually lemma 2 and corollary 2 suggest that reporting should only be observed under competition.

Equation (10) will be chosen if it is incentive compatible for the cartel; equation (10) only satisfies buyers IICC. If the costs are small enough and generate a collusive benchmark price which is smaller than (10) then the cartel will choose the benchmark price, since the benchmark price will not update ξ. While, if the realized costs generate a competitive price that is higher than (10) then collusion is not enforceable in an no-updating strategy. Hence, the next step is to compare equation (10) with the Cournot price, $P^c = \frac{a+nC_{true}}{n+1}$. The comparison generates the following condition:

$$C_{true} \leq \frac{(n+1)P^{m*}_{\sim u} - a}{n} \tag{11}$$

We still need to check whether the no-updating price is binding, preferred to the benchmark collusive. If the benchmark collusive price is smaller than $P^{m*}_{\sim u}$ then the cartel will prefer the benchmark price, since it will no updated buyers cost beliefs, the cartel avoids being reported and at the same time will generate the maximum attainable profits for the cartel. In order to define the region where equation (10) will generate the highest profits for the cartel, without being reported, we will need to set $P^{m*}_{\sim u} \leq P^{m,ben}(C_{true})$.

$$C_{true} \geq 2P^{m*}_{\sim u} - a \tag{12}$$

The above condition defines the minimum value the cost needs to take in order for the cartel to set $P^{m*}_{\sim u}$ and avoid being reported. For values smaller than the above condition the cartel can set the benchmark price and still not be reported. The cartel will not be reported because the cartel will set the benchmark collusive price which will generate higher CS if the buyers do not report. Hence, we have just identified point (B) in the graph. From point (B) and above the cartel will need to satisfy the buyers ICC. The next step is to find a condition to avoid updating ξ, since to derive the above equations we have assumed that ξ is not updated. For this case we need the true cost to be smaller than the maximum anticipated ($C_b \leq C_{max}$), see condition (3). To derive a condition on the maximum prior beliefs, of the buyers, that generates a realized price that is not "suspiciously" anti-competitive we will need to set equation (1) smaller or equal to C_{max}. Therefore, the next condition is derived:

[23] An alternative strategy for the buyers is always not to report but this will not restrain a potential cartel and as a result it always generate the benchmark collusive price. Hence a profitable deviation exists.

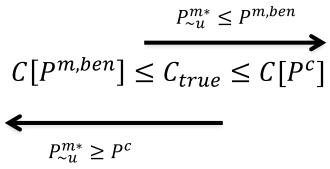
$$C_{max} \geq \frac{1}{2n}[n\xi P - (1-\xi)(2a - 2(n+1)P)] \tag{13}$$

The right hand side of the above equation is strictly positive and smaller than a, suggesting that there is a range for cost beliefs that can generate a price that is competitively rationalizable. Substituting (10) into the above equation we get the following condition.

$$C_{max} \geq P^{m*}_{\sim u} - \frac{a\xi(1-\xi)}{[2(n+1)\sqrt{1+\gamma} + \xi(n+2)]} \tag{14}$$

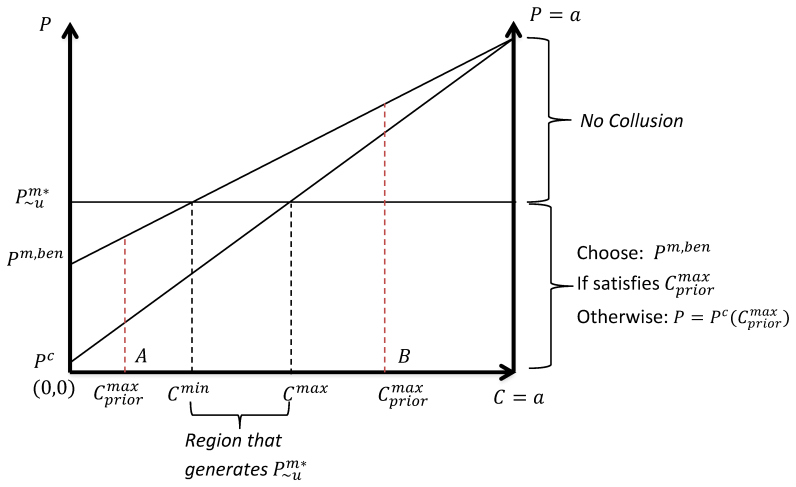
First of all, notice that the right hand side of condition (14) is greater from either (11) or (12), which means that it does not influence the cost region for the ICC of the cartel. To sum up, conditions (11), (12) need to be satisfied in order for the cartel to satisfy its own ICC and condition (14) for the buyers not to update ξ. If the last condition is violated, equation (14), then $P^{m*}_{\sim u}$ is going to update ξ and as a result the cartel should price based on the maximum prior of the buyers. The intuition of the conditions is given below in a figure:

Figure 3: Cartel's ICC



For cost realizations greater than the min point the cartel is not able to set the benchmark collusive price and avoid being reported. For cost realizations smaller than the max point the cartel will be able to price above the competitive price. The next figure presents the pricing schedule of a cartel under different cost realizations.

Figure 4: Pricing schedule under a no-updating strategy



In the above figure we have graphed the price with respect to the cost realization. As the cost increases the prices increases but the difference between collusive and competitive price, as the cost converges to a, decreases. We have plotted the no-updating price, $P^{m*}_{\sim u}$, which is independent from the cost. The no-updating price crosses the collusive price at C_{min} and the competitive at the C_{max}. Prices above $P^{m*}_{\sim u}$ will update the cost and as a result collusion with a no-updating price is not feasible. The region generated between the two cost thresholds can support the $P^{m*}_{\sim u}$, it satisfies ICC of the cartel. For costs smaller than C_{min} the cartel can set the benchmark price, satisfy buyers IICC and avoid being reported. We have also plotted a $C_{max,prior}$, a buyers maximum prior belief of the cost, that is greater than the C_{max} (point B) and smaller than C_{min} (point A). At point (B) the priors do not matter for the cartel's price choice, since the maximum anticipated price is going to be higher than $P^{m*}_{\sim u}$. On the contrary, this is not going to be the case with point (A) since the anticipated price is going to be higher. As a result the cartel will need to choose either the benchmark collusive (if it is ICC for the cartel) or set a price equal to the max prior (again if ICC). Otherwise, the cartel will update ξ, see equation (2). However, in the corollary that follows we establish that point (A) is not possible to happen. The prices that are generated are listed in the lemma that follows. First, define $P(C_{max})$ the price that satisfies the IICC under the prior belief of the buyers (and not using equation 1 in equation 7).

Lemma 4:

The regions that are generated are

A) (14) holds
(11) and (12) holds
Price: $P^{m*}_{\sim u}$

B) (14) holds
condition (11) holds and (12) does not hold
Price: $P^{m,ben}$

C) (14) holds
condition (11) does not hold and (12) holds
Price: - (no-updating price cannot be enforced)

D) (14) does not holds
(11) and (12) holds
1) If $P^{m,ben} > P(C_{max}) > P^c$
Price: $P(C_{max})$
2) If $P(C_{max}) > P^{m,ben} > P^c$
Price: $P^{m,ben}$
3) If $P^c > P(C_{max})$
- (no-updating price cannot be enforced)

E) (14) does not hold
(11) holds and (12) does not hold,

1) If $P^{m,ben} > P(C_{max}) > P^c$
Price: $P(C_{max})$
2) If $P(C_{max}) > P^{m,ben} > P^c$
Price: $P^{m,ben}$

3) If $P^c > P(C_{max})$
Price: P^c

F) (14) does not hold
(12) holds and (11) does not hold,
- (no-updating price cannot be enforced)

Proof: A) is self proven by satisfying the ICC of the cartel, the IICC of the buyers and the prior beliefs of the buyers. As for B), since $P^{m*}_{\sim u} > P^{m,ben}$ the cartel will choose $P^{m,ben}$ because it maximizes its profits. At the same time the buyers will not report since a price that is smaller than the one that makes the buyers indifferent from reporting should generate no reporting as well ($P^{m*}_{\sim u} > P^{m,ben} \Rightarrow CS(P^{m*}_{\sim u}) < CS(P^{m,ben})$). For C) the no-updating price is smaller than the competitive ($P^{m*}_{\sim u} < P^c$ and any price larger than the the no-updating ($P > P^{m*}_{\sim u}$) will update buyers beliefs and make them willing to report if the price does not change, the left hand side of the IICC (equation 7) will be smaller than the rhs. Hence, the cartel will need to set a price that is higher than the no-updating but this will generate reporting. As a result the no-updating price cannot be enforced neither a $P(C_{max})$ exists that satisfy the cartels incentives.

In the fourth case (D) the priors are not sufficiently high to support the no-updating price. If $P^{m,ben} > P(C_{max}) > P^c$ then setting a higher price than $P(C_{max})$ will generate reporting while setting a lower price it will generate smaller profits for the cartel (define $P' < P(C_{max}) < P^{m,ben}$) which means that $\Pi(P') < \Pi(P(C_{max})) < \Pi(P^{m,ben})$ by the concavity of the profit function[24]) and as a result the cartel will choose $P(C_{max})$. If $P(C_{max}) > P^{m,ben} > P^c$ then the cartel can choose the benchmark collusive and avoid being reported. Furthermore, $(P(C_{max})) > P^{m,ben}$ which means that $\Pi(P(C_{max})) < \Pi(P^{m,ben})$, by the concavity of the profit function. Similarly, the cartel will not choose a lower prive and as a result we have the benchmark collusive. Finally, if $P^c > P(C_{max})$ then the cartel cannot be formed. There is no price that can generate higher than competitive profits for the cartel and at the same time avoid be reported.

In case (E) since condition 11 is satisfied then there might be collusive price that will not generate reporting. If $P(C_{max})$[25] is greater than the benchmark collusive then the cartel can choose $P^{m,ben}$ and avoid being reported. If $P(C_{max})$ is smaller than the benchmark collusive but greater than the competitive then the cartel will choose $P(C_{max})$. The IICC under C_{max} is satisfied thus no reporting and the cartel will not lower or raise the price from $P(C_{max})$, see lemma 2. If however $P^c > P(C_{max})$ then the cartel cannot set a price that will satisfy its own ICC, higher profits than competitive, and avoid been reported.

In the last case (F), the no-updating price is smaller than the competitive (since condition 11 is not satisfied). In addition, $P(C_{max})$ is smaller than the no-updating party, which means that the cartel cannot raise the price without being reported. As a result the cartel will not form.

Finally notice that it is not possible for both conditions (11) and (12) not to hold at the same time since condition (12) defines a minimum value for the cost and (11) a maximum (if both conditions do not hold that means that the min is larger than the max).

QED

In the first case (A) all conditions are satisfied and as a result the $P^{m*}_{\sim u}$ can be chosen. The second case states that if the true cost is low enough (12 is violated) then the cartel will choose the benchmark

[24]This is generated by our assumption of linear demand and linear marginal cost.
[25]The price, $P(C_{max})$, will be found by substituting the max prior into the IICC of the buyers.

collusive price which will yield a higher value for equation (7) and the buyers will not be willing to report. The third case states that if the costs are high enough and the no-updating price is smaller than the competitive then collusion is no possible under this strategy. For case (D) a non-updating price can not exist because either ξ will be updated or C. The last case states that if the costs are low enough and the prior of the buyers is low enough as well then the cartel will either choose the benchmark collusive, if the prior on the cost allows to do that, or choose a price that satisfies the prior beliefs of the buyers, if it is incentive compatible for the cartel. A notable result is the fact that all conditions depend on the value of γ. More specifically, as the cost of reporting decreases (γ decreases) the maximum condition decreases as well, which means that it narrows down the range to support a non-updating pricing. On the other hand, for very high cost of reporting the cartel will not need to satisfy buyers IICC and for that reason will choose the benchmark price. However, we still need to examine the case with updating buyers beliefs, in order to find the optimal strategy for the cartel.

B. Collusion, updating strategies in a shockless industry

Let now examine the case where updating is allowed. This is the case where the cartel sets a high price initially and gradually reducing its price until it reaches a steady state point (ss). This will generate high initial prices but lower at the ss compared to the no-updating price. The ss will be lower because of updating ξ. ξ is updated because the cartel will choose to price discriminate across time in a shockless industry and as a result reveal to the buyers its existence. Therefore, the buyers will only be uncertain to the realized cost, that they need to infer in order to decide whether to report (estimate the expected CS under competition). This generates the following law of motion for prices:

$$(a - P_{t+1})^2 = \frac{n^2}{4(n+1)^2}(2a - P_t)^2 - v(a - P_t)^2$$

The above equation has two possible solutions for prices, however there is only one price that generates trade ($P_{t+1} < a$).

$$P_{t+1} = a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t)^2 - v(a - P_t)^2} \tag{15}$$

The above equation needs to hold at any given time period. By lemma 2, as long as the price that satisfies the IICC of the buyers is smaller than the benchmark collusive the cartel will not be reported[26]. If the price defined by equation (15) is larger than the benchmark collusive then the cartel will choose the benchmark price. Furthermore, we can find the steady state (ss) price by setting $P_{t+1} = P_t$.

$$P_u^{m*} = \frac{2a[(n+1)\sqrt{1+v} - n]}{2(n+1)\sqrt{1+v} - n} \tag{16}$$

It is straightforward to show that the above price is smaller than a. When compared to equation

[26]This needs to hold whether $\xi = 1$ or not.

(10) ($P^{m*}_{\sim u}$), we can see that the *no* updating ss price is greater than the above equation (16), as long as $v > 0$ (which holds by assumption). Hence, for the second strategy (updating) to be optimal it must at least hold that the price chosen at period zero (the highest since afterwards its declining) is higher than the unique price set under no updating. However, for the updating strategy to be preferred, the discounted profits under updating have to be are higher than the discounted profits with no updating. Since the cartel will need to enjoy high profits initially and then stabilize at a level lower compared to the case of no updating.

But first of all we need need to find conditions that satisfy the cartels ICCs. The conditions are derived in the same manner with the previous section and they are given below:

$$2P^{m*}_{u} - a \leq C_{true} \leq \frac{P^{m*}_{u}(n+1) - a}{n} \tag{17}$$

Notice that, the both the rhs of the above inequality (the upper bound in C_{true} for the updating ss to be incentive compatible) and the lhs (the min) is smaller than equation (11). Hence we can identify regions of cost realizations that only one strategy is feasible and another that both are feasible.

Lemma 5:

If the true cost belongs to:

$$2P^{m*}_{\sim u} - a \leq C_{true} \leq \frac{(n+1)P^{m*}_{u} - a}{n} \tag{18}$$

Then both strategies are feasible. If the true cost takes values between:

$$2P^{m*}_{u} - a < C_{true} < 2P^{m*}_{\sim u} - a$$

Then the no-updating strategy with the benchmark price is feasible. Finally, if it takes value between:

$$\frac{(n+1)P^{m*}_{u} - a}{n} < C_{true} < \frac{(n+1)P^{m*}_{\sim u} - a}{n}$$

Then only the updating strategy is enforceable.

Proof: Condition (18) has been derived by comparing conditions (11), (12) and (17). This condition is necessary and sufficient for both strategies to be feasible. It is necessary and necessary since there is no other region that satisfy cartel's incentives, buyers IICC and both strategies are feasible. If however the second condition (defined in the lemma) holds then the updating ss price is going to be larger than the benchmark collusive ($P^{m*}_{u} < P^{m,benc}$) but this does not hold for the no-updating. This means that the cartel can set the benchmark collusive from the very first period keep that price and avoid be reported. If the cost belongs to the third region then the no-updating price is smaller than the competitive but the updating strategy leads to a price larger than the competitive and as a result only the updating strategy is enforceable ($P^{m*}_{\sim u} < P^{c}$).

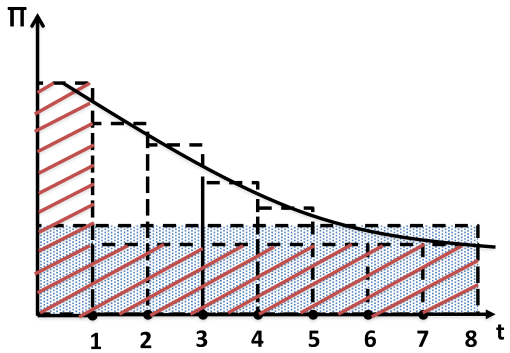
QED

The above condition states that for ss prices that are close to the benchmark price the no updating price generates higher profits (the maximum value of C_{true} in the no-updating is higher). The main

reason is the tradeoff between the choice of no-updating, and profiting from the buyers uncertainty on ξ, and from the choice of profiting in the short run from the uncertainty on the cost of the cartels. In other words, if the ss price of a no-updating strategy is close to the benchmark price then the cartel will not profit much if price discriminating across time and update, as a result, the buyers beliefs and as a result it will be optimal for the cartel to choose a constant price across time. However, notice that the lower bound of the above condition is more binding (higher) for the no-updating strategy. This means that for low costs the updating strategy is preferable.

In order to find the optimal strategy, in the region that both strategies are feasible, we will need to find which strategy generates the highest stream of profits and whether there is a strategy that always dominates. However, to compare the stream of profits we will need to define the pricing path for the updating strategy. This task is not infeasible, since we have the law of motion and the starting price needs to be at most the benchmark. The price needs to be at most the benchmark one, since an updating strategy will need to accumulate as high profits as soon as possible since it drops to lower profits in the long run (compared with the no-updating strategy). However, there is one last condition that both strategies need to satisfy and that is the cartel stability condition. To be more specific both strategies will need to satisfy a minimum discount factor (MDF) for a cartel to be stable (deviation not be profitable). This mainly creates issues with the latest strategy since a tradeoff is generated. The cartel will need to be impatient to choose the updating strategy (accumulate high profits in the short run) but patient enough to make deviation unprofitable. As for the latest strategy, a further problem is created with respect to the discount factor. Since the price might have infinitely changes until it converges to the ss which will generate infinite roots for the discount factor (when we try to identify a MDF that satisfies both stability and the updating strategy). However, we are able to show existence, rather than uniqueness, which is actually what is needed in our case. We do that by assuming that the updating strategy correspond to setting the benchmark price in the first period and the ss price afterwards, in contrast to infinite price changes. Hence, if there is a MDF that generates stability and higher profits, in the above discrete case, than the no-updating case then necessarily it should also hold for the general case as well. An illustration of the argument is presented in the figure below.

Figure 5: Sufficient conditions.



The blue region, corresponds to profits that are generated by a no-updating price. These profits are constant across time and necessarily higher than the ss profits of the updating strategy. We have also plotted the profits for the updating strategy across time (decreasing squares), where we have assumed that the profits converge to the ss within five periods (this was made for illustration purposes. Finally, the red shaded area corresponds to the profits, for the updating strategy, that we are going to examine. If a discount factor exists which generates higher profits, for the red shaded area, that are higher than the blue area then this discount factor needs to generate higher profits for the updating strategy with

more than one period of high profits. The conditions are presented below:

$$\delta \geq \frac{\Pi^{d} - \Pi^{m,ben}}{\Pi^{d} + \Pi_{u}^{m*} - \Pi^{m,ben} - \Pi^{c}} \tag{19}$$

$$\delta \leq \frac{\Pi^{m,ben} - \Pi_{\sim u}^{m*}}{\Pi^{m,ben} - \Pi_{u}^{m*}} \tag{20}$$

Equation (19) refers to the stability condition of the updating strategy. For the updating strategy to be ICC with no deviation the MDF needs to be higher than the rhs of (19), the cartel needs to be patient enough. While equation (20) refers to the discount factor required for the updating strategy to be more profitable than the no-updating. In the latter case the cartel members need to be sufficiently impatient. The proposition that follows presented necessary conditions for an updating strategy to be optimal. We

Proposition 1:

> Assume that the priors defined by equation (14) hold. An updating strategy to be optimal then it is sufficient for the condition (17) to hold and (19) and (20) if condition (18) holds.

Proof: For the buyers not to report at the ss and for the cartels ICC to be satisfied then condition (17) needs to hold. If however condition (18) holds at the same time (a region of it at least) then both the updating and the no-updating strategy might be optimal. However, if conditions (19) and (20) then the optimal strategy is the updating. To be more specific conditions (19) and (20) are derived in order for a revised form of the updating strategy to be optimal. Where revised we refer to the cartel setting the benchmark collusive price at period one and the ss updating price from period 2 and onwards. This price schedule necessarily generates lower profits than gradually dropping the price. Thus, for a price schedule defined $A = P_0^{m,benc}, P_1', P_2', ..., P_{u,T}^{m*}$ and $B = P_0^{m,benc}, P_{u,1}^{m*}, ..., P_{u,T}^{m*}$ with $P_1' > P_{u,1}^{m*}, P_2' > P_{u,2}^{m*}$ it generates a stream of profits for A that is higher than B; $A = \Pi(P_0^{m,benc}) + \delta\Pi(P_1') + \delta^2\Pi(P_2') + ... + \delta^T\Pi(P_{u,T}^{m*}) + .. > B = \Pi(P_0^{m,benc}) + \delta\Pi(P_{u,1}^{m*}) + \delta^2\Pi(P_{u,2}^{m*}) + .. + \Pi(P_{u,T}^{m*}) + ...$ This means that for the minimum value of δ that satisfies the ICC of the cartel under price schedule B then that value of δ will satisfy for A. Furthermore, since the present value of A is higher than B that means that lower values, than the ones defined by (19) and (20), of δ can be supported. Hence, we define sufficient but not necessary conditions since there might be smaller values of delta under price schedule (A) that will make the updating strategy optimal.

If the cost is higher than the rhs of (17) then the ss of the updating price is lower than the competitive price which means that the cartel has a finite span of existence and as a result there will be no cartel (by a backwards induction argument). If the cost is lower than the lhs of (17) then the ss price is larger than the benchmark price and as a result the cartel will not choose the updating strategy. Any initial price will be higher than the ss price, by definition of the updating strategy, which will generate initially lower profits. That however, violates the optimality of the updating strategy, since the cartel will choose the no-updating and generate higher collusive profits.

Finally, if assumption (14) does not hold then another set of regions will need to be examined, see for example lemma 4.

QED

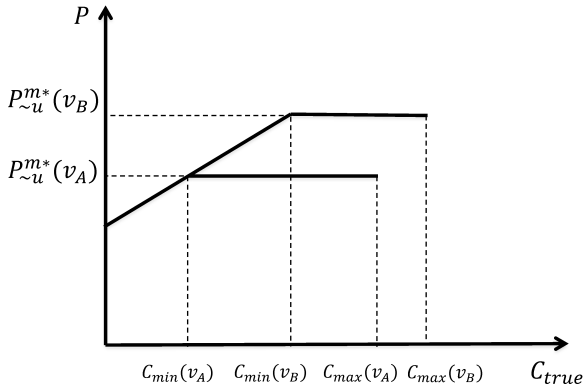
Furthermore, if an MDF such that the optimal strategy is the updating one exists then the rhs of (19) needs to be greater than the rhs of (20). After some simple algebra and we have reached to the following simplified form:

$$(\Pi^{d} - \Pi^{m,ben})(\Pi^{m*}_{\sim u} - \Pi^{m*}_{u}) < (\Pi^{m,ben} - \Pi^{m*}_{\sim u})(\Pi^{m*}_{u} - \Pi^{c}) \tag{21}$$

We have calibrated the above condition and found that it is satisfied for the following values: $a = 100, n = 3, v = 0.01$, $\xi = 0.55$ and $C_{true} = 10$. These parameter values support the updating strategy as the optimal one when the discount factor takes values between $(0.825, 0.859)$.

Assume now, that the discount factor is such that it does not allow the buyers to use the updating strategy. Assume as well that the prior beliefs do not influence the no-updating equilibrium, equation (14) holds. Under these assumptions the collusive price should be equal to the benchmark price for cost realizations smaller than the lhs of equation (12), equal to the ss of the no-updating price for costs between (12) and (11) and no collusive price for costs greater than the rhs of (11). Furthermore, notice that the $P^{m*}_{\sim u}$ is positively related to the cost of reporting (v), condition (11) is also influenced positively but by half the impact to the collusive price, while the impact on condition (12) is positive is also greater than the one in $P^{m*}_{\sim u}$. These results are presented below:

Figure 6: No-updating optimal price choices under different cost realizations



In the above figure we have plotted the collusive price with respect to the cost. The price is a function of the cost of reporting with $v_A < v_B$. As we can see the range that $P^{m*}_{\sim u}$ is enforceable shrinks as the cost of reporting increases. In other words, it is easier for the cartel to set the benchmark collusive price. Moreover, as the v increases the maximum price that the cartel can set also increases (upper bound of price). This is because of buyers unwillingness to report which restrains their ability to update their information on the realized cost of their suppliers. Finally, for very high cost realizations the cartel is not able to set a price that will avoid updating buyers beliefs and for that reason a collusive price is not enforceable. The updating is not applicable either since its upper bound is smaller than the one with no-updating. In other words, the ss price under updating will converge to a price that is smaller than the competitive and as a consequence the cartel will dissolve. Hence, for small costs realizations the suppliers are more able to collude while at high cost realizations it becomes more and more difficult to collude up to the point where it is impossible.

C. Introducing a shock

Let now assume that the parameters support an updating strategy[27]. Let also allow for a temporary *publicly* known shock to hit the industry at period 1 and at the ss. The shock is realized at the next period (for example at $t = 2$ if the shock was introduced in $t = 1$). The choice of a publicly known shock is taken in order to resemble the ones to the retail gasoline industry where the crude oil prices are publicly known and influence future prices rather than current prices (the prices do not adjust immediately). Our assumption of an industry with no shocks does restrain us from comparing our predictions with the empirical predictions of the rock and feathers literature. The next section, where the cost is stochastic, will be more relevant to the retail gasoline industry. Furthermore, we assume that buyers memory spans only for one period. As we are going to show, allowing for more memory will have a greater impact on the prices.

The equations used to derive the impact of the shock to the price path are a modification of equation (15). Notice that the impact of the shock is different depending on its direction. The reason is that a shock is informative. This will be more explicitly explained in the next proposition.

Proposition 2:

In a non-stochastic industry, with lack of memory for the buyers, a positive shock will update both the upper and lower bound of the cost for the buyers. While a negative shock will only update the upper bound. The updated equations are the following:

$$P_{t+1} = \begin{cases} a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t - 2 * |u_t|)^2 - v(a - P_t)^2} & \text{if } u_t > 0 \\ a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t + |u_t|)^2 - v(a - P_t)^2} & \text{if } u_t < 0 \end{cases} \tag{22}$$

Proof: First of all, the lack of memory establishes that a change in buyers beliefs will take place at the period of the shock only (by assumption). Furthermore, a positive shock shifts both the lower and upper bound upwards since the probability of $C^b < u_t$ is zero, otherwise the shock would not have occurred, hence the lower bound is updated to u_t. The upper bound is updated since by definition it is a positive shock and as a result the max MC will be equal to the current belief of the max MC plus the shock ($C_{max,t+1} = C_{max,t} + u_t$). Equation (22) with a positive shock is derived by using the buyers IICC and substituting for ($E(C_t^B) = \frac{P_t + 2 * u_t}{2}$), see equation (7), the error term is multiplied by two since both the lower and the upper bound shifts by u_t.

A negative shock shifts downwards the upper bound only since the $Pr(C_t < 0) = 0$. To be more specific, a negative shock cannot update the lower bound since the costs cannot be negative (bounded by below). The upper bound is updated since the $Pr(C_{t+1} > C_t - u_t) = 0$, the MC is decreased and the upper is decreased as well. Equation (22) with a negative shocks depends only on u_t (and not twice the shock as with the positive case) since only the upper bound is updated.

QED

Where u denotes the shock. The shock influences the IICC with respect to the competitive alternative. The buyers will decide to report if the competitive price, which is a function of the expected cost having added the shock, is smaller than the expected collusive price. As a result the buyers need

[27]We have chosen to introduce a shock under the updating strategy since otherwise the impact of the shock would have an immediate jump to a new ss price which will not generate an adjustment path.(NEED TO EXAMINE THIS MIGHT GET INTERESTING RESULTS.)

to take in consideration the impact of the cost shock in the case where the suppliers are going to compete and the information that a shock can provide to the possible realization of the true cost. Notice that equation (22) describes different behaviour of the cartel under different shocks. Furthermore, we have multiplied u_t with 2 under a positive shock because the upper bound will be equal to $P_t + u_t$ and the lower bound will be u_t and as a result $E(C_t^B) = \frac{P_t+2*u_t}{2}$. While for a negative shock the $E(C_t^B) = \frac{P_t-u_t}{2}$, since only the upper bound is updated. Hence, the cartel will be able to change its price by a higher magnitude (almost twice as much) after a positive shock rather than a negative shock. The asymmetric behaviour, that arises, is a consequence of the buyers' learning process. Notice that the lack of memory means that the buyers from the next period and onwards will anticipate a price adjustment as with equation (15)[28]. Furthermore, the fact that after the shock we revert to equation (15) that means that the ss price will stay unchanged.

Most importantly, a positive shock has double the effect on prices than a negative shock. Finally, the following figure presents a calibrated case of our results.

Figure 7a: Shock at the adjustment path

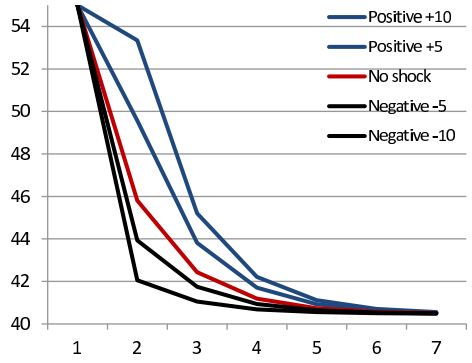
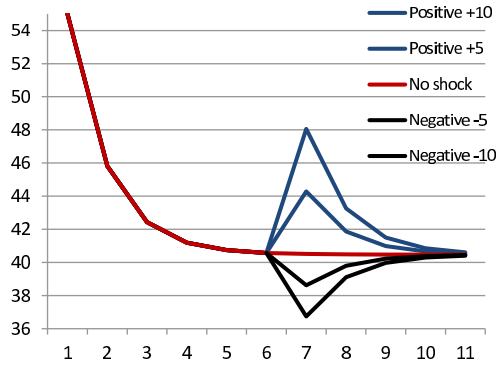


Figure 7b: SS shock



In the vertical axis we have plotted the prices and in the horizontal axis the time periods. In both figures we have assumed that the starting price of the cartel is the benchmark price, which is 55. The competitive price, under the parameters used, is 27.5. The calibration values are the same with the ones used to show the existence of parameters that generate the updating strategy as an optimal for the cartel to take. In the first period of the shock we used equation (22), while from the second period and onwards we used equation (15) to derive the above graph. Notice that the convergence to the ss (the red line in both graphs) is achieved within a relative short period of time, approximately 5 periods. In figure (7a) we have introduced a shock at the second period of the adjustment path, while at (7b) we have introduced the shocks in a period where the price has achieved its ss value. The blue lines correspond to a temporary increase of cost (positive shock of 5 and 10) and the black lines correspond to a temporary decrease on the cost (negative shock of 5 and 10). The shocks are chosen such that they satisfy the cartel's ICCs.

From the first graph we can see that the price stabilizes faster (price differences from the ss are much smaller) when the shock is negative and when the shock is introduced in the second period. This also holds if the shock is introduced at the ss the length of adjustment is the same. However, as we are going to show persistence weakens (but still exists) when we relax the assumption of no memory. Most importantly, the prices react asymmetrically to the shocks. There is a greater change after a positive shock than under a negative shock. The actual changes are the following, for graph (7a) the net

[28]We implicitly assume that the buyers forget that a shock ever existed but they remember what the price in the next period should be. In other words, when the shock hits at t the buyers expect a P_{t+1} while when the time period $t+1$ arrives the buyers forget about the shock but remember what price they were expecting (P_{t+1}).

change is 3.76387 for a positive shock of 5 while 1.88124 for a negative shock of an equal magnitude, with a net difference of 1.8826. For the biggest shock, the change is equal to 7.52997 for a positive shock while 3.76207 for a negative shock, with a net difference of 3.7679. The changes are reported in net value and were calculated based on a common reference price, the price without a shock at t=2. Similar results hold for (7b); the price increase, after a positive shock on the costs, is greater than the price decrease. The reason for the asymmetry is that the price is influenced positively and nonlinear on the cost and because of buyers learning process where a positive shock is more informative than a negative.

Another interesting result is that a temporary shock at the ss (figure 7b) will generate a series of changes (and not an unique change), but it still converges to the same ss. Thus, our results predict a persistence of the shock and persistence to the asymmetries, the changes remain asymmetrically different. For example a shock of 5 generates a net asymmetry of 1.8859 in the 1st period, 0.6925 in the 2nd, 0.2541 in the 3rd and 0.0932 in the fourth, where with net asymmetry we mean the change after a positive shock minus the change after a negative shock (with the reference price being the ss price). The asymmetries are generated by the buyers incentive to report. The introduction of the shock makes the buyers more willing to report if the shock is negative (they anticipate a low competitive price). But because of the lack of memory the buyers become less willing to report and the price converges back to its initial ss. The opposite reasoning holds if the shock is positive. The buyers become less willing to report since their future alternative (competition) will generate a smaller benefit, but since the lack of memory the buyers become more willing to report and as a result the price drops gradually to its initial ss. The impact of the cost of reporting and the benefits of reporting being enjoyed in a future date is that the cartel can postpone and profit from a slow convergence to the ss.

To sum up, we find asymmetric changes, persistence, importance on the magnitude of the shock and higher volatility for positive shocks (the price changes are relatively greater). The intuition is the following, with a temporary positive shock the cartel will have an incentive to raise the price as much as possible but it is constrained by the threat of reporting, it needs to satisfy the buyers IICC. While a negative shock makes the cartel less willing to share the benefits of the savings on the cost. The cartel also benefits from the ability to span the changes across time, a result of the cost of reporting and the lack of memory. The span though seems symmetric but the volatility of a negative shock is smaller than the volatility of a positive shock.

Let now relax the assumption of no memory and assume that buyers remember all price changes (the next results hold even if we assume that the buyers remember only the two latest prices). The shocks will update the lower and upper bound, on the cost of the buyers, in different way depending on its sign. In the first period updating takes place in the same manner as before. However, a shock is informative, even temporary in nature, and as a result will have permanent effects to the buyers beliefs and as a consequence to the ss price. The last argument is presented in the proposition that follows.

Proposition3:

> *A negative temporary shock will permanently update the upper bound, while a positive temporary shock will also permanently update the upper bound if $n \geq 4$. Additionally, a negative temporary shock will permanently update the lower bound as well.*

Proof: The immediate impact of the shock is the same as with proposition 2. However, in the next period, when the MC returns to its initial value, the shock will permanently update the lower bound under a negative shock. This is easy to show, by taking the derivative of equation (22) with respect

to u_t when $u_t < 0$ then:

$$\frac{dP_{t+1}}{du_t} = \frac{n^2}{4(n+1)^2} \frac{(2a - P_t - |u_t|)}{\sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t + |u_t|)^2 - v(a - P_t)^2}}$$

Setting the above derivative smaller than 1 then $\frac{4(n+1)^2-n^2}{4(n+1)^2}(2a - P_t - |u_t|)^2 - v(a - P_t)^2 > 0$ which always holds, otherwise there is no price in real numbers. Hence, the price decrease is smaller than the actual magnitude of this shock. This means that the $Pr(C_{true,t+1} > P_t - u_t) = 0$ and as a consequence the upper bound is updated. The lower bound is updated as well since the MC returns to its initial value and the $Pr(C_t < u_t) = 0$ otherwise the MC would have been negative.

If there is a positive shock, however, the lower bound is not updated even after the reversion to the initial MC, see proposition 2. The upper bound is updated if the price that the cartel chooses is smaller than the previous price plus the shock, $P_{t+1} < P_t + u_t$. Taking the derivative of (22) with respect to u_t when $u_t > 0$ we have the following equation:

$$\frac{dP_{t+1}}{du_t} = \frac{n^2}{2(n+1)^2} \frac{(2a - P_t - |u_t|)}{\sqrt{\frac{n^2}{4(n+1)^2}(2a - P_t + |u_t|)^2 - v(a - P_t)^2}}$$

The difference with the previous equation is that the impact of the shock is twice as much. Using the same approach as with the negative shock, we reach the following condition:

$$\sqrt{2n+1}(n - \sqrt{2v(n+1)}(a - P_t) + n\sqrt{2n+1}(a - 2u_t) > 0$$

If $n \geq 4$ then the above condition is satisfied. This is a sufficient condition since there might be values of v such that even if $n < 4$ the condition will hold. As a result if $n \geq 4$ then $P_{t+1} < P_t + u_t$ which means that P_{t+1} is informative to the upper bound of the MC.

QED

The 3rd proposition shows that in the second period, the negative shock updates both the lower bound and the upper bound of the buyers' cost beliefs, in contrast to the first period. The negative shock suggests that there is zero probability for the true cost to take a value smaller than the magnitude of the shock (otherwise the cost at the time of the shock would have been negative, something that we have excluded by assumption). Hence, the lower bound is updated and is equal to the shock. At the same time the upper bound will be updated since the price after the shock is smaller than the initial price plus the shock. As a result the upper bound (C_{max}) will be equal to the price after the shock minus the absolute value of the shock.

However, this is not the case with a positive shock. In the second period the lower bound of the distribution is not updated but only the upper bound is updated, the cost returns to its initial value and $Pr(C^B < u_t) > 0$. The upper limit will be updated if the price after the shock is smaller than the price before the shock having added the shock. As we have argued in the previous paragraph the price after the shock is always smaller and as a result the max bound will be equal to the new price minus the shock.

Notice that compared to the initial impact of the shock a positive shock is less informative from the second period and onwards than a negative one. This is because of the permanent nature of the shock. As for the updated values of the MC, these are:

$$C^B_{t+2}\begin{cases}\epsilon[0, min(P_t+u_t, P_{t+1})-u_t] = \frac{P_{t+1}-u_t}{2} & \text{if } u_t > 0\\ \epsilon[u_t, min(P_t-u_t, P_{t+1})+u_t] = \frac{P_t}{2} & \text{if } u_t < 0\end{cases}$$

Where, t is the time period that a shock has been realized. Moreover, from the 3rd period and onwards the updating reverse to its initial process. Hence, from the 3rd period the updating process is:

$$C^B_{t+3+i}\begin{cases}E[0, min(C^B_{t+2+i}, P_{t+2+i})] & \text{if } u_t > 0\\ E[u_t, min(C^B_{t+2+i}, P_{t+3+i})] = \frac{P_t}{2} & \text{if } u_t < 0\end{cases} \tag{23}$$

Where i takes values from zero to T, the time periods after the shock. While the law of motion becomes, for a positive shock (if $u_t > 0$):

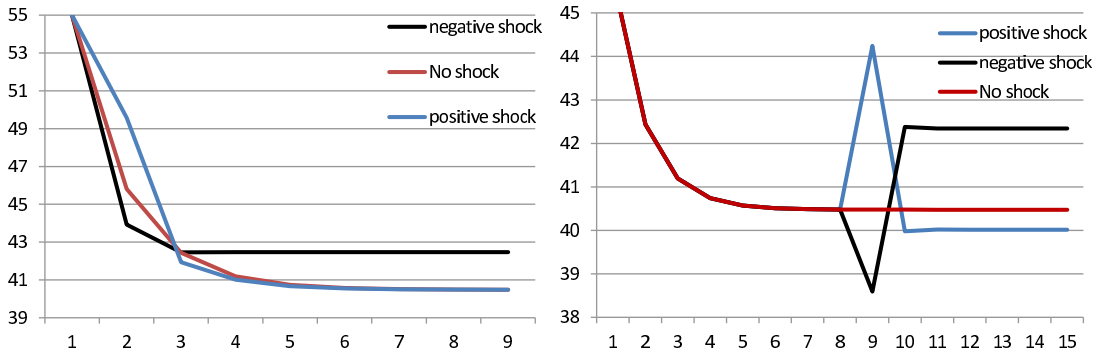
$$P_{t+3+i} = a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - min(C^B_{t+2+i}, P_{t+2+i})))^2 - v(a-P_{t+2+i})^2} \tag{24}$$

For a negative shock (if $u_t < 0$):

$$P_{t+3+i} = a - \sqrt{\frac{n^2}{4(n+1)^2}(2a - min(C^B_{t+2+i}, P_{t+3+i}) - u_t)^2 - v(a-P_{t+2+i})^2} \tag{25}$$

In the graph that follows we have plotted the impact of a temporary, both a positive and a negative, shock to the price path in two different phases, near the ss and during the adjustment path. Notice that a shock is informative to the buyers, either to the max or min bound, and generates a smaller region for the realized cost to belong to. Interestingly a negative shock will generate high ss price if the shock is realized at the adjustment period, while a positive shock will converge to a lower ss price (which is going to be the same with the case where there was no shock). The reason is that a shock at the adjustment path can only be informative with respect to the lower bound in the long run, in the short run information of the upper bound will be obsolete since the prices are dropping and the maximum bound is updated. However, a positive shock does not update the lower bound in the long run and as a result a positive shock has no real effect in the long run (at the ss prices). Notice that the negative shock is more informative, than a positive shock, but lowers the CS (more information is not preferred in this case). Furthermore, if the shock is realized at a period where the price has reached its ss value then the negative shock will have no real impact on the ss price (it will converge back to the initial ss), since the lower and upper bound are updated by the same magnitude. However, this is not the case with a positive shock; under a positive shock only the max bound is updated and as a result the new ss will be lower than the initial. Consequently, in either case a positive shock has a welfare enhancing result to the buyers, in some cases short term and in others long term. The next figure presents this case.

Figure: 8



In contrast to figures (7a) and (7b) the shocks in this case are less short lived, within a period the prices are quite close to their ss value. Furthermore, in this figure (8) we can see that there is a real effect of the shocks depending on whether the shock was positive or negative and whether it was realized at a transitionary phase or at the ss value. Notice that the asymmetric behaviour is also present in figure (8). The ss prices are going to be different and the initial price change is going to be greater in magnitude after a positive shock rather than a negative. The net difference in the first period is 1.8826 and 1.8828 at the ss when the shock happened at the adjustment phase, while the net difference when the shock is realized at the ss prices is 2.32578. The permanent (asymmetric) difference is -1.9912 for a shock at the adjustment phase and -0.4802 for a shock at the ss. Where the difference is defined as $DA = (P_t^+ - P_{ss}) - (P_t^- - P_{ss})$, where the sign corresponds to a positive or negative shock, ss to steady state and DA asymmetric difference. Interestingly, this suggests that the asymmetries reverse from a point and afterwards. For example, a positive shock will have a positive impact on prices in the first period but a negative impact on the second period. This is driven by the fact that the positive shock generates a lower ss irrespectively of the timing of the shock, even though the magnitude differs. A positive shock updates the upper bound and generates a more optimistic expectation on the real cost, while a negative shock updates only the lower bound and as a result generates a pessimistic expectation on the realized cost (expected cost increases)[29]. However, a positive shock is not informative, at the Long run, if the shock is realized at the adjustment path (at t=1) since the upper bound will be updated in any case and as a result there is no effect at the ss price. Hence, the cartel will be able to price higher after a negative shock rather than a positive shock.

The key assumption for the permanent effect of the shocks and the decrease in the magnitudes of the persistence (differences from the new ss become very small quite quickly) is the buyers memory. If buyers compare prices across a longer time period then they are more informed and the cartel cannot postpone the updating process. Moreover, the shocks even though temporary reveal information on the suppliers realized cost. To sum up, we find asymmetric changes on prices, persistence, reversal of the impact of the shock to the prices (cyclical behaviour) and a permanent effect of a temporary shock.

The next task is to plot the impact to the pricing after a permanent shock. A permanent shock cannot be informative with respect to the upper bound (instantaneously at least), since the costs are not anticipated to change. A positive permanent shock is informative with respect to the lower bound, $Pr(C^B < u_t) = 0$, however this is not the case with a negative shock. Furthermore, we are going to examine two scenarios; the buyers memory is bounded to one period in one of the cases, they do know that the shock is permanent though, and in the other case the buyers have full memory. In the first case the upper bound of the cost will be equal to $min\{P_t, P_{t-1}\}$ and in the latter case $min\{P_t, P_{t-i}\}$

[29]In other words, the buyers alternative of collusion is becoming less desirable after a negative shock.

where i expands to all the periods after the shock. The results are depicted in the following graphs.

Figure 9a: Permanent Shock at the ss, restricted memory

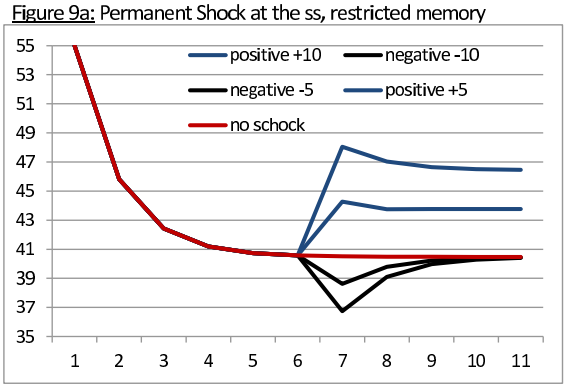


Figure 9b: Permanent Shock at the ss with full memory

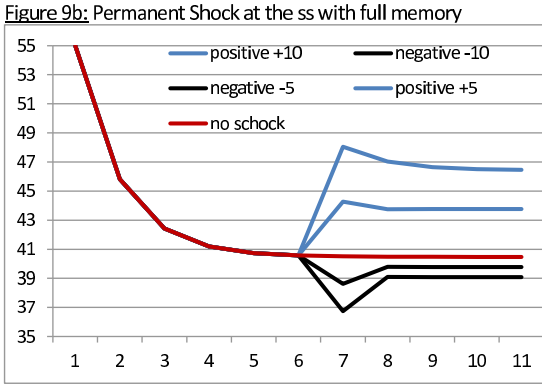


Figure (9a) presents the case where the buyers have limited memory. The fact that a positive shock revises the lower limit upwards has as a result to generate a higher ss price. This is because the buyers become less optimistic (expected cost is higher) for the competitive price and the benefits of reporting. While this is not the case with a negative shock, after the realization of the shock the lower bound is not updated. Interestingly the price overshoots in the first period and then it adjusts to the new ss. The overshooting is present independently of the direction of the shock. The reason for the adjustment and the correction of the price is the incentive of the buyers to report, which increases after the overshooting. Furthermore, the buyers lack of memory has as a result for prices to return at their initial ss under a negative shock. Hence, in this case the cartel can manipulate buyers belief to generate a better ss price. However, this is not the case if the buyers have full memory, figure (9b), since the buyers remember what was the minimum highest price from the time that a shock was implemented and as a result manipulating the beliefs is not feasible.

As for the asymmetries, the initial price changes are greater after a positive shock rather than a negative shock. Furthermore, the memory matters for the long run magnitude of the asymmetries. Under full memory the asymmetries are greater at their ss, see figure (9b). Hence, in this case the persistence of the asymmetries is independent of the memory but the size (magnitude) is not. The intuition for the asymmetries is the following; if the shock is negative the cartel is willing to pass through as less as possible of the cost savings. However, the buyers update their beliefs on the expected cost and as a result are more willing to report. The cartel will decrease the price to a smaller amount than the ss because the buyers are faced with reporting cost. On the other hand, if the shock is positive the cartel will want to pass through as much as possible of the cost increase and the fact that buyers update both the lower and upper bound of the distribution helps the cartel to increase its price even further. Most importantly, we can see that the generated prices are sticky and shocks generate a real effect on prices irrespectively of whether the shock is temporary or permanent. When the shocks hits the industry the prices overshoot but then they converge to a ss slowly. Furthermore, the volatility of a positive shock will be greater irrespectively of the base used to calculate it, whether we use the initial or the final ss price. Finally, the variance of prices in a collusive regime is going to be greater than the variance of competitive prices. This is straightforward to see, since the derivative of equation (22) is going to be greater than the derivative of the competitive price with respect to the cost. Notice that the last result is not be driven by the fact that the cartel chooses to reveal its existence (updating strategy) since in the no-updating strategy the cartel members will need either to mimic the behaviour of Cournot or switch to an updating strategy, in either case the variance will be greater or equal to the one under competition. This result is in contrast to Harrington and Chen (2006) who find that collusive prices will exhibit lower variation. A further comparison will be taken

in the next section were the shock is stochastic.

The above results suggest that prices asymmetrically respond to a shock might be a result of collusive behaviour. The rocks and feathers literature suggest that capacity constraints, see Maskin and Tirole (1988), and search costs, see M. Tappata (2009) and H. Yang and L. Ye[30], can explain the asymmetric responses to shocks in a competitive setting. However, the capacity constraint models might not be appropriate to a retail model. Furthermore, gasoline stations are located quite close to each other (some of the times within 50 meters of distance) and as a result the search costs argument might not be appropriate in this industry. Additionally, the literature does not find reversal of the asymmetries something that is present in empirical studies. As for papers that refer to collusive behaviour, Green's and Porter's (1984) paper has been widely been referred as a model that can explain asymmetries. However, the paper's regime switch strategy is the driving force of the asymmetries, while in our paper asymmetries emerge naturally through buyers' learning. Furthermore, their model might be inappropriate to explain the persistence of the asymmetries that have been found by empirical work, see for example S. Borenstein, et al. (1997).

As a robustness check the next section presents the case where suppliers compete with respect to prices, Bertrand competition, and demand is inelastic.

IV. Bertrand Competition and inelastic demand

Inelastic demand, with Y being the maximum willingness to pay and S the market size. Competition is taking place with respect to prices, *à* la Bertrand. A monopolist will choose a price equal to Y while under competition the price will be equal to the true marginal cost, C. The C is realized from a uniform distribution with a minimum support equal to zero and maximum a finite value which is less that Y (in order for trade to take place). The buyers do not know the true costs of their suppliers. However, learning is possible by eliminating regions of costs based on the observed prices, as with the previous section. To be more specific, if the suppliers compete then the price should be fully informative ($C^B = P$, where B is for buyers) while if the suppliers collude the price is not fully informative. A collusive price updates the upper bound of the cost beliefs but the true cost can be anything between zero and the price. This means that the upper bound of the cost, $\bar{C}^B$, will be equal to the price. To sum up, the cost, from the buyers point of view, will be equal to:

$$\begin{cases} C_t^B = P_t & \text{if competition,} \\ E_t(C_t^B) = \frac{P_t}{2} & \text{if collusion.} \end{cases}$$

Furthermore, the buyers do not know whether the buyers are colluding or not and as a result the expected cost will be equal to:

$$E_t(C_t^B) = \xi_t \frac{P_t}{2} + (1 - \xi_t) P_t \tag{26}$$

For the above equation to be rationlizable, the price does not reveal a cartel (and as a result the expected cost is a weighted average between the competitive and the collusive case), the updated value needs to be smaller than the prior belief on the max support of the cost. As a result:

[30]Other papers include work by Lewis (2011); and Cabral and Fishman (2006) among others.

$$\bar{C}_t^B = min\{E_t[C_t^B], \bar{C}_{t-1}^B\} \tag{27}$$

As for ξ, this will be the same with equation (3) of the previous section. The IICC, though, will change because of the inelastic demand assumption. The new IICC is given below:

$$\xi_t(Y - P_{t+1}^m) + (1 - \xi_t)(Y - E_t P_{t+1}^c) = Y - E_t P_{t+1}^c - v(Y - P_t)$$

Taking expectations with respect to the price at $t + 1$:

$$P_{t+1}^m = \frac{1}{\xi_t}[vY - P_t(v - \xi_t + \frac{\xi_t^2}{2})] \tag{28}$$

Notice that with inelastic demand the variance terms do not appear, which means that the buyers are not interested on second order effects to their CS. As with the previous section, the above equation relates future prices to current. The next step is to examine the cartel strategies.

A. Cartel Strategies under Bertrand Competition and Inelastic Demand

The two strategies available to the cartel are the updating and the *no* updating. The equilibrium price in the *no* updating strategy will be found by setting $P_{t+1}^m = P_t$ in equation (28). The steady state (ss) price is:

$$P_{\sim up} = \frac{2vY}{2v + \xi_{prior}^2} \tag{29}$$

The above equation is generated based on the incentives of the buyers not to report. Furthermore, the cartel members will not have an incentive to lower their price since it will only reduce their profits. At the same time the cartel members will not choose a higher price since that will generate updating, the buyers will need to be faced with lower price in the future in order for them not to report (which will generate updating), which contradicts the initial assumption of a *non* updating strategy.

As for the updating strategy, the equilibrium ss price is:

$$P_{up} = \frac{2vY}{2v + 1} \tag{30}$$

In the above equation we have imposed $\xi = 1$ since the prices will change across time and update buyers beliefs. Notice that equation (29) is always smaller than (30) for a strictly positive ξ_{prior}. Hence, the *no* updating strategy generates a higher ss price[31] which means that the cartel needs to be relatively impatient to use the updating strategy.

Furthermore, the above equations suggest that if the true costs are greater than the rhs of (30) but

[31]For any value of ξ_{prior} which is weakly greater than zero but weakly smaller than 1.

smaller than the rhs of (29) then the no updating strategy is going to be the only feasible. If the cost is greater than the rhs of (30) the cartel will be able to extract high collusive profits but the ss price will be lower than the competitive. As a result the cartel has a finite date of existence, since the price becomes smaller or equal to the competitive, then by the argument of deviation the cartel cannot be sustained in any period. If the realized cost is greater than the rhs of (29) then neither the updating nor the no updating strategy is feasible. In either case the cartel cannot set a price that will be stable across time, in order to avoid be reported. If, however, the cost is smaller than the rhs of (30) then both strategies are feasible at the same time. The results are summarized below:

- If $C > \frac{2vY}{2v+\xi^2_{prior}}$ then no collusion
- If $\frac{2vY}{2v+1} < C < \frac{2vY}{2v+\xi^2_{prior}}$ then the *no* updating strategy is optimal.
- if $C < \frac{2vY}{2v+1}$ then either of the strategies might be optimal.

The above conditions hold for appropriate priors of the buyers (BE MORE EXPLICIT DESCRIBE ALL THE REGIONS IN A PROPOSITION). As with the previous section, in order to find the optimal strategy, we will need to find which strategy generates the highest stream of profits and whether there is a strategy that always dominates. To compare the stream of profits we will use the same approach. The conditions are the same with the previous section. However, because of the simplifying assumptions taken in this section we are now able to show existence. Hence, the next inequality needs to hold for the updating strategy to always dominate the no-updating one.

$$\xi_{prior} > \frac{2v(n-1)(Y-C)}{2vn(Y-C)-C} \tag{31}$$

The right hand side of (31) is going to be smaller than 1 if:

$$v > \frac{C}{2(Y-C)} \tag{32}$$

Which in turn, it is required that:

$$C < \frac{2Y}{3} \tag{33}$$

Let me remind the reader that C is for the true cost of the suppliers. The last condition states that for the updating strategy to be optimal the cost of the suppliers is required to be low enough. At the same time condition (32) states that the cost of reporting needs to be high enough. Notice that high cost of reporting allows the suppliers to extract higher profits, as a result condition (32) states that suppliers need to be profitable enough. While conditon (31) states that the buyers need to be suspicious enough for the updating to be optimal. In other words, if the buyers are already very suspicious on the competitiveness of the industry (high ξ_{prior}) then the cartel will find it more profitable to update the buyers' beliefs. High ξ_{prior} reduces the profitability of the no updating strategy and as a result it makes the updating strategy more attractive. Conditions 31-33 state that there are

parameter values where the updating policy is optimal, all three need to be satisfied. Our next task is to introduce a shock to the industry.

B. *Introduction of a shock*

Let the shock, u_t be generated from a Uniform distribution with min support $-B$ and maximum B, where $B < Y$. Let also the shock be publicly know and the parameters are such that the cartel will choose the updating strategy. The introduction of the shock will change the expectations of the buyers on the competitive price change, equation (28), in the following manner:

$$P_{t+1}^{m} = \frac{1}{\xi_t}[vY - (P_t + u_t)(v - \xi_t + \frac{\xi_t^2}{2})] \tag{34}$$

First of all, we are going to explore the case with a temporary *publicly* known shock that hit's the industry at period 1 and at the ss. The impact of the shock is realized at the next period (for example at $t = 2$ if the shock was introduced in $t = 1$). As for the impact of the shock on the buyers information; a positive shock shifts both the lower and upper bound, on the beliefs of their suppliers cost, upwards (for example $pr(C_t^b < u_t) = 0$ in a positive shock) while a negative shock shifts downwards the upper bound only. For example, if the buyers believe that the cost takes value between zero and 10 and a positive shock of 2 happens in the industry then the cost will need to take values between 2 and 12 with a mean of 7. If the cost is negative then only the upper bound is updated (the cost cannot be negative, -2) and it takes values between 0 and 8 with a mean of 4. As a result, when a temporary shock is realized the upper and lower bound are updated to:

$$C_t^B \text{ E } U[max(0, u_t), P_t + u_t] \tag{35}$$

The lower bound will be equal to the maximum between zero and the shock. This means that if the shock is positive the lower bound will be equal to the value of the shock, while if it is negative the lower bound will be equal to zero. The upper bound will be equal to the current price plus the shock. If a negative shock is realized the upper bound is reduced while under a positive shock the upper bound increases. However, it is always possible for the cartel to choose a price that is less than the last periods price plus the shock $P_{t+1} < P_t + u_t$ and as a result the cartel will further update buyers beliefs. Furthermore, a temporary negative shock has a permanent impact to the lower bound. This is mainly because the negative shocks reveals that the realized shock could not have been lower than the value of the shock ($Pr(C_t^B < |u_t|) = 0$). However, this is not the case with a positive shock, since after the implementation of the shock to the prices then the lower bound should be equal to zero (the probability of the cost being less than the value of the shock is not zero). The updating process of the lower and upper bound for the period after the shock is given below:

$$C_{t+1}^B \text{ E } U[max(0, -u_t), min(P_t, P_{t+1} - u_t)] \tag{36}$$

Notice that the lower bound is equal to the maximum between zero and minus the shock ($\underline{C}_{t+1}^B = max(0, -u_t)$, where $\underline{C}^B$ is the buyers belief on the lower bound of the cost). This representation captures the fact that if there is a negative shock then the lower bound should be updated upwards,

while if there is a positive shock the lower bound should be equal to zero. The upper bound after a positive shock will be equal to the minimum between the price before the shock was implemented and the price observed with the implementation of the shock minus the value of the shock (the costs return to their initial value), $min(P_t, P_{t+1} - u_t)$. The intuition is simple, it is always possible that the cartel might choose not to pass-through all the cost savings to the buyers which means that the upper bound should be equal to the initial price. In our example, if the cartel has chosen a price of 9 then the upper bound should be 10 and not 11 (after the costs return to their initial values). In the case of a positive shock the lower bound will remain at zero and the upper bound will be equal to the minimum between the price before the shock and the price after the shock minus the shock. In this case the cartel might choose not to increase its price to the full value of the shock (in order to avoid be reported). In our example it is possible the cartel to raise its price to 11, instead of 12, and since the shock is temporary then the new price reveals information on the buyers cost, the cost should be 9 (11-2). From the second period and onwards the updating becomes:

$$C^B_{t+2} \text{ E } U[max(0, -u_t), min(P_{t+2}, min(P_t, P_{t+1} - u_t))] \tag{37}$$

To sum up, a negative shock updates only the upper bound in the first period while it updates the lower bound from the second period and onwards. We are going to show that the upper bound is not updated since the cartel will not set a price that is lower than the initial price minus the cost. A positive shock in the first period updates both the lower and the upper bound, while in the second period and afterwards only the upper bound is updated. This means that a negative shock is initially, in the first period, less informative than a positive shock but more informative from the second period and onwards. This means that the low of motion will be different under a positive or negative shock. The equations when the shock first hits are provided below:

Under a positive shock (both lower and upper bound are updated):

$$P^m_{t+1} = vY - (P_t + 2u_t)(v - \frac{1}{2}) \tag{38}$$

Under a negative shock (only upper bound is updated):

$$P^m_{t+1} = vY - (P_t + u_t)(v - \frac{1}{2}) \tag{39}$$

The updating process generates asymmetric responses on prices. A positive shock will have twice as much impact to the prices than a negative shock. In our example a positive shock of 2 increases the expected mean to 7, a net change of 2 (from the initial expected mean), while a negative shock of 2 decreases the expected mean to 4, a net change of 1. From the second period and onwards the law of motion becomes:

Under a positive shock (only upper bound is updated):

$$P^m_{t+2} = vY - (\bar{C}^B_{t+2})(v - \frac{1}{2}) \tag{40}$$

While for a negative shock (both upper bound and lower bound are updated)

$$P^m_{t+2} = vY - (\underline{C}^B_{t+2} + \bar{C}^B_{t+2})(v - \frac{1}{2}) \tag{41}$$

The above set of equations generate the same qualitative effects to the pricing schedules of the cartel with the previous section.

V. Industry with a stochastic shock

In this section we will keep the assumptions of Bertrand competition and inelastic demand. We are now going to assume that the cost of the suppliers exhibits a stochastic term. The true cost is realized at the birth of the industry (period zero), which is not know by the buyers, and from that period and onwards a publicly known shock is present which will be realized at the next period (this is going to be the main difference with the Harrington and Chen paper. Otherwise the analysis is going to be the same; their law of motion (price path) is an AR(1) process, as in this paper, and the buyers learn on the parameters of the distribution. A major difference is the assumption of the authors that buyers form beliefs on price changes rather than cost changes and that they take as given the buyers incentives of the buyers to report. Finally, I have also decided to focus on asymmetric responses while Harrington and Chen focus on the variance of price when suppliers compete or collude.). Our assumptions define the following cost function: $MC = C + u_t$, where MC is for Marginal cost. Let the shock be random and generated from a Uniform distribution with minimum support $-A$ and maximum A. The buyers observe the shock with an error, which means that the buyers believe that $u_t \sim U[-B, B]$. The fact that the buyers observe the shock with an error means that $|A| < |B|$. Alternatively, this assumption suggests that buyers believe that the shock will have a greater impact (whether positive or negative) to the cost structure of their suppliers. Intuitively if the buyers observe the crude oil shocks they anticipate that the cost of the retailers will be proportionally higher independently of the direction of the shock. We could have assumed instead that the impact of the shock is greater if the shock is positive and smaller if it is negative, but then the asymmetries would have been influenced or magnified even further (the asymmetries would have been driven by our assumptions as well). Furthermore, the fact that the buyers observe the shock with an error means that any value between -B and B can be anticipated.

Let assume that the buyers have a memory of one period (actually this assumption generates an AR(1) process on updating and as a result it is sufficient to keep track of all the history). The buyers learn about the constant fraction of the MC from the realizations of the shocks. To be more specific, the lower bound at t will be equal to the minimum between minus the shock at t and the lower bound at $t-1$ ($\underline{C}^B_t = min(\underline{C}^B_{t-1}, -u_t)$, where $\underline{C}^B_t$ is for the buyers beliefs on the lower bound of the cost). In other words, a sequence of negative profits will update the lower limit, this is the same reasoning with the previous section; for a negative shock to be realized then $Pr(C < u_t) = 0$, where C in this case is the constant term of the MC. On the other hand, a sequence of positive shocks should not update the lower limit since the probability of C being smaller than a positive shock is no zero. The upper bound will be equal to the minimum between the updated upper bound of the previous period and the maximum between the current price minus the previous period shock and the previous price. The updating process is provided below:

Min of the constant term of the MC:

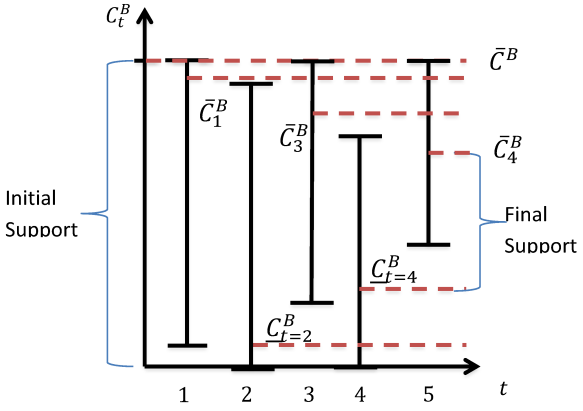
$$\underline{C}^B_t = max(\underline{C}^B_{t-1}, -u_t) \tag{42}$$

Max of the constant term of the MC (the current price minus the positive shock will give the min maximum bound (of the upper bound) of the constant term while the current price plus the absolute value (minus the negative shock) will give the max upper bound as well) :

$$\bar{C}_t^B = min(max(P_t - u_{t-1}, P_{t-1}), \bar{C}_{t-1}^B) \tag{43}$$

The above equations describe the buyers beliefs formation, which are going to be used to infer the cost of the suppliers. The buyers are willing to learn about the costs in order to form expectations on the competitive price and decide whether to report. The next figure presents the intuition of the above process.

Figure 10: Updating process with a stochastic shock



In the above figure we have plotted the buyers beliefs on the support for the constant term (C_t^B) at the vertical axis and time at the horizontal axis. We have introduced a positive shock in periods 1 and 3 and a negative shock in periods 2 and 4. In order to simplify the representation we have assumed that the shocks are fully observed. We have also abstract from the information that a price might provide, this was done in order to simplify the representation of the updating process. A shock shifts the distribution downwards or upwards. However, a positive shock will not shift the the distribution above the upper bound (it is bounded by the maximum willingness to pay, a condition for the market to exist) and as a result it updates the upper bound by the value of the shock (see point $\bar{C}_1^B$ and the red dashed line that tracks the change to the bounds). If a negative shock is observed then the lower bound is updated, it is not possible after a negative shock to observe negative MCs and as a result a negative shock updates the lower bound. This means that the support of the distribution shrinks, the length of the lines (that correspond to the support of the cost distribution) becomes smaller and smaller over time.

The next task is to describe the updating process on the likelihood of collusion (ξ). The likelihood of collusion in this case should be equal to the probability of the cost (both the constant and the stochastic part) being smaller or equal to the observed price. This is equivalent to to examine the cdf of the observed price from the updated cost distribution. As a result a high price is more likely to be collusive and suspicious rather than a low price. The likelihood is given below:

$$\xi_t = \frac{P_t}{\underline{C}^B_{t-1} + \bar{C}^B_{t-1}} \tag{44}$$

Finally, we examine the IICC of the buyers and define the price path that will make the buyers indifferent from reporting. The shocks are going to influence the part of equation (5) that has to do with buyers making inference on future prices. We can rewrite equation (5) in the following way:

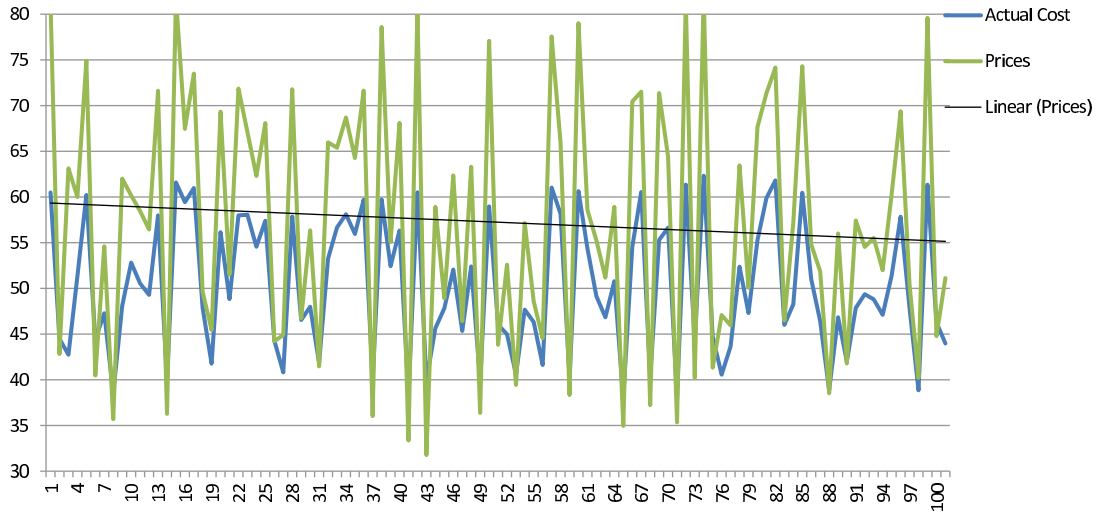
$$P^m_{t+1} = \xi_t E_t(C^B_{t+1}/\xi_t = 1) + (1-\xi_t)E_t(C^B_{t+1}/\xi_t = 0) \tag{45}$$

The expected competitive price (Bertrand) is going to be equal with the constant fraction of the MC (C) plus the shock at t which will be realized at $t+1$. As a result the competitive price at $t+1$ should be the price at t minus the the expected value of the shock at $t-1$, which will give C under competition, plus the expected value of the shock at t; $E_t P^B_{t+1} = P_t - \frac{u_{t-1}}{2} + \frac{u_t}{2}$. If however the suppliers are colluding then the buyers will need to infer the future price based on their beliefs of the min and max bound of the cost and the shock at t. However, it is always possible for a big negative shock and as a result we should incorporate that possibility: $\frac{max[\underline{C}^B_t + u_t, 0] + \bar{C}^B_t + u_t}{2}$ the shock is added twice in this case since it shifts both the lower and the upper bound. As a result the law of motion, it was derived in the same approach as with previous sections. The main difference is that the expected cost is conditional to the new information at t and its history. The equation is provided below:

$$P^m_{t+1} = \xi_t \frac{(\bar{C}^B_t + max[\underline{C}^B_t + u_t, 0])}{2} + u_t + \frac{vY}{\xi_t} - (1-\xi_t)\frac{u_{t-1}}{2} - P_t(\frac{v}{\xi_t} - \frac{(1-\xi_t)}{2}) \tag{46}$$

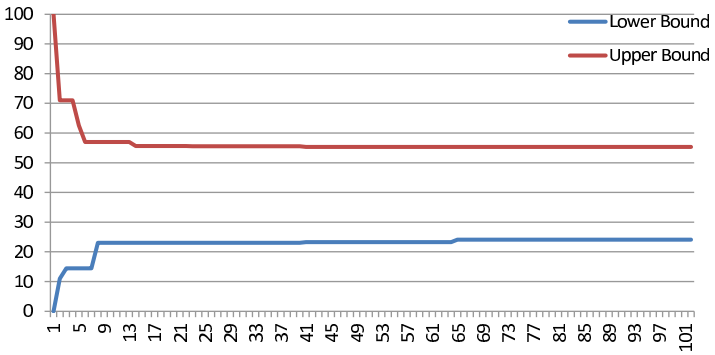
We have calibrated our results by setting $Y = 100$, uniform iid shocks with $B = 25$, $A = B/2$, $v = 0.2$ and $C = 50$. We have assumed that in the first period the suppliers were competing and in the second period they have colluded. The prices are given in the following graph.

Figure 11:



On average the cartel members have managed to raise prices by 6.20 and we had only two cases where the suppliers have actually suffered damages (the damages though were less than -0.5). Furthermore, we can see the prices exhibit greater variance than the shocks. This is contrast to Harrington's and Chen's result. The reason for the difference is that the suppliers cannot manipulate buyers beliefs with respect to the shocks behaviour, in their model the buyers know that the industry is characterized by a random component in the cost but do not know the actual direction of the shock and its actual magnitude. In addition, it will be interesting to see how the beliefs are updated across time. This is presented in the next figure.

Figure 12:



As we can see, both the lower and upper bound are updated quite fast, within 10 periods. The lower bound reaches approximately 25 and the reason for that is the magnitude of the shocks, the maximum negative shock is -25, which means that the lower bound cannot be further updated. Moreover, the upper bound reaches a long run value close to 55, this means that the suppliers by satisfying the IICC and pricing with a lower margin provide more information to the buyers than what they would wish for. Notice though that the buyers expect a constant MC of 40 (average value) and as a result the buyers beliefs are biased downwards since the actual cost is 50. Hence, the buyers are over-optimistic with respect to the constant fraction of their suppliers MC.

As for the asymmetries, the introduction of frequent shocks make it intractable to examine the cartel's response to positive and negative shocks. However, we use S. Borenstein, A. C. Cameron and R. Gilbert (1997) reduced form to estimate them (IT MIGHT SHOW THIS BY TAKING THE DERIVATIVE WITH RESPECT TO PRICE AND FINDING THE PERSISTENCE PARAMETER) . To be more specific we regress the differences of prices to the positive shocks and negative shocks, to their lags, to the lag difference of prices and two lags of the difference of ξ. The equation is:

$$\Delta P_t = \beta_0 + \beta_1 u^-_{t-1} + \beta_2 u^+_{t-1} + \beta_3 lag\Delta P_{t-1} + \beta_4 \Delta\xi_{t-1} + \beta_5 lag\Delta\xi_{t-1} + \varepsilon_t \tag{47}$$

Where u^-_{t-1} is the shock in the previous period and takes the value of the shock if the shock was negative and zero otherwise, u^+_{t-1} is the shock in the previous period and takes the value of the shock if the shock was positive and zero otherwise. The results of the regression are presented in the following table.

Table 1:

Source	SS	df	MS	Number of obs =	98
				F(11, 86) =	7969.56
Model	50214.0185	11	4564.91077	Prob > F =	0.0000
Residual	49.2602139	86	.572793185	R-squared =	0.9990
				Adj R-squared =	0.9989
Total	50263.2787	97	518.178131	Root MSE =	.75683

pricedif	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lag1negvalue	.8282366	.0584505	14.17	0.000	.7120409	.9444324
lag1posvalue	.9229813	.0534615	17.26	0.000	.8167034	1.029259
lag2negvalue	-1.195403	.055961	-21.36	0.000	-1.30665	-1.084156
lag2posvalue	-1.233403	.0609968	-20.22	0.000	-1.354661	-1.112146
lag1pricedif	.3113134	.0747703	4.16	0.000	.1626749	.4599519
lag2pricedif	.0049368	.0129051	0.38	0.703	-.0207176	.0305912
lag3posvalue	.3916941	.0594749	6.59	0.000	.273462	.5099262
lag3negvalue	.3389494	.0564894	6.00	0.000	.2266522	.4512465
difxi	-57.0829	3.702804	-15.42	0.000	-64.44383	-49.72197
lag1difpos~2	-.0036167	.0021992	-1.64	0.104	-.0079886	.0007552
lag1difneg~2	-.0005601	.0023738	-0.24	0.814	-.005279	.0041589
_cons	-.2539258	.3034596	-0.84	0.405	-.8571835	.3493318

We have tested a hypotheses whether cost differences of more than 3 lags and price differences of more than 2 lags should be used in the regression and rejected it. As we can see, the asymmetries exist and persist across time. To be more specific, the prices change by 12% more after a positive shock rather than a negative; the coefficient of negative shock is .82824 while the coefficient of a positive shock is .92298. Furthermore, the asymmetries persist for two more period, the lag value is statistically significant, but the difference reverse in sign for the second lag but not for the third. Moreover, the lags suggest that prices initially overshoot and then correct to the new values (this is because of the sign reversal). Most importantly, the asymmetries do not reverse in magnitude; in other words the positive shock have a greater impact on future prices as well. Interestingly, the effects of the shock are greater in the second period than the first period. However, the sum of all three lag coefficients is positive which means that the impact of the shock is spanned into more than one period but there is a net impact on the prices (even though the shocks are temporary). Furthermore, we have included the square of the shock but it seems that it is not statistical significant, which means that the magnitude of the shock does not matter. Hence, we have found asymmetries and of up to two periods persistence.

S. Borenstein, A. C. Cameron and R. Gilbert (1997) have found that the price change is twice as much after a positive rather than a negative shock, in the retail gasoline market. They found persistence, with the sign reversing in the lag values only for the negative shock. Interestingly, they have found that the net change after a negative shock is a positive increase on prices (the sum of the coefficients between the first shock and the lag is positive). Finally, their asymmetries do seem to reverse after the third period (in net values). The table with S. Borenstein, A. C. Cameron and R. Gilbert (1997) results is given below.

Table 2:

TABLE I
ESTIMATES OF PRICE ADJUSTMENT EQUATIONS

Downstream price:	Retail	Spot gas	Terminal	Terminal	Retail
Upstream price:	Crude	Crude	Spot gas	Crude	Terminal
Periodicity:	Semi-monthly	Weekly	Weekly	Weekly	Semi-monthly
Observations:	164	351	351	351	164
$\Delta\text{Upstream}^{+}_{0}$	0.549	0.888	0.593	0.558	0.623
	(0.084)	(0.135)	(0.075)	(0.063)	(0.069)
$\Delta\text{Upstream}^{+}_{-1}$	0.246	0.691	0.041	0.178	0.357
	(0.087)	(0.140)	(0.058)	(0.062)	(0.083)
$\Delta\text{Upstream}^{+}_{-2}$	0.022	−0.161	0.058		−0.101
	(0.088)	(0.136)	(0.048)		(0.079)
$\Delta\text{Upstream}^{+}_{-3}$					−0.024
					(0.076)
$\Delta\text{Upstream}^{-}_{0}$	−0.181	1.088	0.182	0.210	0.199
	(0.136)	(0.143)	(0.081)	(0.071)	(0.120)
$\Delta\text{Upstream}^{-}_{-1}$	0.236	−0.239	0.145	0.203	0.251
	(0.098)	(0.127)	(0.048)	(0.052)	(0.080)
$\Delta\text{Upstream}^{-}_{-2}$	0.028	−0.286	−0.001		0.065
	(0.100)	(0.116)	(0.040)		(0.080)
$\Delta\text{Upstream}^{-}_{-3}$					−0.133
					(0.070)
$\Delta\text{Downstream}^{+}_{-1}$	−0.314	−0.055	0.180	0.289	−0.507
	(0.123)	(0.096)	(0.090)	(0.077)	(0.109)
$\Delta\text{Downstream}^{+}_{-2}$	0.069	−0.009	−0.203		0.174
	(0.119)	(0.098)	(0.089)		(0.107)
$\Delta\text{Downstream}^{+}_{-3}$					0.004
					(0.095)
$\Delta\text{Downstream}^{-}_{-1}$	0.127	−0.070	0.310	0.332	−0.086
	(0.131)	(0.094)	(0.092)	(0.077)	(0.112)
$\Delta\text{Downstream}^{-}_{-2}$	0.396	0.279	0.042		0.271
	(0.119)	(0.091)	(0.081)		(0.087)
$\Delta\text{Downstream}^{-}_{-3}$					0.103
					(0.077)
Upstream_{-1}	0.141	0.131	0.229	0.081	0.278
	(0.040)	(0.037)	(0.034)	(0.020)	(0.054)
Downstream_{-1}	−0.175	−0.183	−0.254	−0.118	−0.262
	(0.045)	(0.037)	(0.039)	(0.024)	(0.054)
Time	0.225	0.326	0.011	0.165	−0.015
	(0.109)	(0.113)	(0.044)	(0.056)	(0.069)
R^2	0.663	0.589	0.646	0.630	0.916

Notice that in the table the above upstream prices refer to crude oil prices, for the retail market (the first column). The second column is the spot gas market and the shocks are the crude oil prices, the third column they present the terminal market and the shocks are the the spot gas prices. In the fourth column they present the results of the terminal market with respect to crude oil shocks and in the final column they present results from the retail market when the shocks are the terminal prices. The authors find asymmetric response to cost changes, persistence and in some cases reversal of the sign in higher lag differences. But the impact of a positive shock dominates the impact of a negative.

Our calibration results seem to much the results of this paper pretty well. We do find asymmetries, persistence of up to three periods for the cost shocks, reversal of the sign and a positive shock dominating the effect of a negative shock at the long run (sum of the coefficients). However, we fail to match the exact size of the asymmetries. To be more specific, the authors find that the price changes, for the first lag shock, is double as much for the positive shock while in our calibration exercise we have found a difference of only 12%. Notice that in the previous sections where the shocks were not

as frequent (industries without a shock and then introducing a shock) we have actually shown that the asymmetries will be double in difference.

Finally, if we exclude ξ_t from the regression then the asymmetries increase, it actually doubles to 25%. The persistence does not change but the effect of the second lag of the shocks is smaller than the first lag and there is reversal of the asymmetries as with S. Borenstein, A. C. Cameron and R. Gilbert (1997). Finally, the net effect of a positive shock is an increase on prices (when we sum up the coefficients of all the lags of the shocks) and a decrease on the prices after a positive shock. In other words, when we exclude ξ we can better replicate the qualitative results of the authors.

VI. Conclusion

This paper examines the strategic interaction of buyers and suppliers when the threat of collusion between suppliers exists and the buyers are able to learn suppliers' realized cost and likelihood of collusion. We show that the threat of reporting enforces the cartel members to satisfy the buyers' IICC. The cartel is profiting from both buyers' uncertainty and their cost of reporting, unwillingness to report. We have shown that the uncertainty increases the cost of reporting in two ways. First of all, it decreases the future anticipated gains for the buyers. Higher anticipated costs decrease the benefits from the industry switching to competition, higher anticipated competitive price. At the same time it increases the cost of reporting, parameter γ. For higher cost it is more likely that the cartel is competing and as a result the buyers become less willing to report.

In a shock-less economy the cartel has two available strategies. One of them is to reveal its existence, updating strategy, and the other one to operate secretly, no updating strategy. For the updating strategy to be optimal the MDF is required to be small enough, while at the same time the cartel's stability requires the MDF to be high enough. However, we show the existence of parameters that generate the updating strategy as the optimal one. In the updating strategy the cartel behaves self-destructively. It sets high initial prices, generating high profits in the first periods, with the trade off of lower ss prices, relative to the no-updating strategy.

Most importantly, we find asymmetric response of the cartel with respect to positive and negative cost shocks. The cartel will choose to pass-through as much as possible of the cost's increases to the buyers while it will pass-through as less as possible of the cost's savings. This is mainly because the cartel needs to satisfy the buyers IICC which restrains the ability of the cartel to set the benchmark collusive price and the learning process of the buyers. Consequently, cost shifters give the opportunity to the cartel to increase its margin. In addition we find persistence of the asymmetries across time, lower volatility under a negative shock, sign reversal, which means that prices behave cyclically, and a real effect on prices after a temporary shock. Moreover, our results are robust to different assumptions. We have also found that price stickiness depends on the frequency of the shocks. Finally, as far as we know the empirical literature has not examined the asymmetry in volatility and the magnitude effect of a shock. This is left for future research.

VII. References

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VIII. Appendix

A. Nonzero Collusive variance

The IICC is:

$$E_t[(a - P^m_{t+1})^2] = E_t[(a - P^c_{t+1})^2] - \frac{v(a - P_t)^2}{\xi}$$

Which can be decomposed to:

$$[E_t(a - P^m_{t+1})]^2 + Var(a - P^m_{t+1}) = [E_t(a - P^c_{t+1})]^2 - \frac{v(a - P_t)^2}{\xi} \tag{48}$$

The collusive variance by the assumption of a uniform distribution is $Var(P^m_t) = \frac{(P_t)^2}{12}$. This is derived by using the fact that the maximum collusive price will be is $\frac{a+C_{t,max}}{2}$ and the minimum is

$\frac{a}{2}$. Hence, using the assumption that the cost is generated by a uniform distribution and substituting into equation (48):

$$[E_t(a - P^m_{t+1})]^2 + = [E_t(a - P^c_{t+1})]^2 - \frac{v(a - P_t)^2}{\xi} - \frac{(P^m_t)^2}{12} \tag{49}$$

The above equation generates a new IICC:

$$P^m_{t+1} = a - \sqrt{\frac{[(2a - P_t)(n + 1 - \xi) + n\xi P_t]^2}{4(n + 1)^2} - \frac{v(a - P_t)^2}{\xi} - \frac{(P^m_t)^2}{12}} \tag{50}$$

The difference with equation (8) and (9) is that the last term at the rhs of the above equation, $\frac{(P^m_t)^2}{12}$, appears. Therefore, it becomes easier for the cartel to raise its price. To be more specific, an increase on the rhs of the above equation will generate an increase on the collusive price. The intuition is that the uncertainty on the collusive price generates as second order effect that allows the cartel members to raise their prices. This is driven by the incentives of the buyers. As the buyers become more uncertain on what will the collusive price be then they are less willing to report.